

# STRATFORD PUBLIC SCHOOLS

## Stratford, Connecticut



*“Tantum eruditi sunt liberi”*  
Only The Educated Are Free

**Statistics 2 Curriculum**

**Adopted by the Board of Education**

**January 2018**

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## **DISTRICT MISSION**

The mission of the Stratford school community is to ensure that all students acquire the knowledge, character and 21st century skills to succeed through high quality learning experiences and community partnerships within a culture of diversity and respect.

## **DISTRICT CORE VALUES**

Students will acquire content knowledge, strengthen higher-order thinking, and develop character in order to address 21st century challenges.

## **BUNNELL HIGH SCHOOL BELIEFS**

We believe teachers must work collaboratively in support of student learning and to model collaboration as a social skill with students. We believe that a rigorous curriculum for all students, an acceptance of diversity, and a culture that actively welcomes all learners will contribute to a more knowledgeable community and society. We believe in the value of a strong education as a means of preparing students for work and life in the remainder of the 21st century.

## **STRATFORD HIGH SCHOOL BELIEFS**

- a safe, positive school climate that embraces diversity is essential to ensure respect and opportunity for each individual
- students should understand the world beyond their community in order to contribute to a global society
- parents and students must share responsibility and work in partnership with the school in order to improve academic performance and to develop lifelong learners
- students should use technology effectively to acquire, process, and deliver information

## **BUNNELL HIGH SCHOOL and STRATFORD HIGH SCHOOL**

### **LEARNING EXPECTATIONS**

All students will...

- use real-world digital and other research tools to access, evaluate and effectively apply information appropriate for authentic tasks. (Academic)
- work independently and collaboratively to solve problems and accomplish goals. (Civic-Social)
- communicate information clearly and effectively using a variety of tools/media in varied contexts for a variety of purposes. (Academic)
- demonstrate innovation, flexibility and adaptability in thinking patterns, work habits and working/learning conditions. (Academic)
- effectively apply the analysis, synthesis and evaluation processes that enable productive problem solving. (Academic)
- value and demonstrate personal responsibility, character, cultural understanding and ethical behavior. (Civic-Social)
- show competence in all core academic subjects and other fields of interest, including the ability to clearly and effectively communicate content information in multiple formats. (Academic)

## STATISTICS PACING GUIDE

Unit Name and Synopsis	Projected # of Days	Actual # of Days	Factors that Affected the Pace of Learning	Implications for Curriculum and Unit Design
<b>Unit 1: Chapter 6</b> <i>Confidence Intervals</i> <b>In this unit the focus will be on:</b> <ul style="list-style-type: none"> <li>➤ <b>Confidence Intervals for the Mean</b> <ul style="list-style-type: none"> <li>○ Large Samples</li> <li>○ Small Samples</li> </ul> </li> <li>➤ <b>Confidence Intervals for the Population Proportions</b></li> </ul>	3.5 weeks			
<b>Unit 2: Chapter 7</b> <i>Hypothesis Testing with One Sample</i> <b>In this unit the focus will be on:</b> <ul style="list-style-type: none"> <li>➤ <b>Introduction to Hypothesis Testing</b></li> <li>➤ <b>Hypothesis Testing for the Mean</b> <ul style="list-style-type: none"> <li>○ Large Samples</li> <li>○ Small Samples</li> </ul> </li> <li>➤ <b>Hypothesis Testing for the Population Proportions</b></li> </ul>	4.5 weeks			
<b>Unit 3: Chapter 8</b> <i>Hypothesis testing with Two Samples</i> <b>In this unit the focus will be on:</b> <ul style="list-style-type: none"> <li>➤ <b>Testing the Difference between Means</b> <ul style="list-style-type: none"> <li>○ Large Independent Samples</li> <li>○ Small Independent Samples</li> <li>○ Dependent Samples</li> </ul> </li> <li>➤ <b>Testing the Difference between Proportions</b></li> </ul>	5 weeks			
<b>Unit 4: Chapter 9</b> <i>Correlation and Regression</i> <b>In this unit the focus will be on:</b> <ul style="list-style-type: none"> <li>➤ <b>Correlation</b></li> <li>➤ <b>Linear Regression</b></li> <li>➤ <b>Measures of Regression and Prediction of Intervals</b></li> </ul>	5 weeks			

**STATISTICS 2 UNIT PLANS # 1 - 4**  
**Stratford Public Schools Statistics**  
**Unit # 1**

**Unit Name:** Chapter 6 *Confidence Intervals*

**Est. # of Weeks:** 3.5 weeks

**In this unit the focus will be on:**

- **Confidence Intervals for the Mean**
  - Large Samples
  - Small Samples
- **Confidence Intervals for the Population Proportions**
- **Confidence Intervals for Variance and Standard Deviation**

**STUDENT LEARNING GOALS**

**CCSS Math Standards**

**CCSS.MATH.CONTENT.HSS.IC.A.1**

Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

**CCSS.MATH.CONTENT.HSS.IC.B.3**

Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

**CCSS.MATH.CONTENT.HSS.IC.B.4**

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

**CCSS.MATH.CONTENT.HSS.IC.B.6**

Evaluate reports based on data.

**CCSS.MATH.CONTENT.HSS.ID.A.2**

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more data sets..

**CCSS.MATH.CONTENT.HSS.ID.A.3**

Interpret differences in shape, center, and spread in the context of the datasets, accounting for possible effects of extreme data points (outliers).

**CCSS.MATH.CONTENT.HSS.ID.A.4**

Use the mean and standard deviation of the data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

**21<sup>st</sup> Century Skills and Expectations**

**Rubric: Critical Skills**

1. Use real-world digital and other research tools to access, evaluate, and effectively apply information appropriate for authentic tasks.
2. Work independently and collaboratively to solve problems and accomplish goals.
3. Communicate information clearly and effectively using a variety of tools/media in varied contexts for a variety of purposes.
4. Demonstrate innovation, flexibility, and adaptability in thinking patterns, work habits, and working/learning conditions.
5. Effectively apply the analysis, synthesis, and evaluative processes that enable productive problem solving.
6. Value and demonstrate personal responsibility, character, cultural understanding and ethical behavior.

<p><b><u>Math Practices</u></b></p> <p><a href="#">CCSS.Math.Practice.MP1</a> Make sense of problems and persevere in solving them.</p> <p><a href="#">CCSS.Math.Practice.MP2</a> Reason abstractly and quantitatively.</p> <p><a href="#">CCSS.Math.Practice.MP3</a> Construct viable arguments and critique the reasoning of others.</p> <p><a href="#">CCSS.Math.Practice.MP4</a> Model with mathematics.</p> <p><a href="#">CCSS.Math.Practice.MP5</a> Use appropriate tools strategically.</p> <p><a href="#">CCSS.Math.Practice.MP6</a> Attend to precision.</p> <p><a href="#">CCSS.Math.Practice.MP7</a> Look for and make use of structure.</p> <p><a href="#">CCSS.Math.Practice.MP8</a> Look for and express regularity in repeated reasoning.</p>	<p><b><u>Interdisciplinary Standards (Technology Integration)</u></b></p> <p><b>Standard 1: Information Strategies</b> Students determine their need for information and apply strategies to select, locate, and access information resources.</p> <p><b>Standard 2: Information Use</b> Students evaluate, analyze, and synthesize information and data to solve problems, conduct research, and pursue personal interests.</p> <p><b>Standard 3: Information and Technology Application</b> Students use appropriate technologies to create written, visual, oral and multimedia products that communicate ideas and information.</p> <p><b>Standard 4: Literacy and Literary Appreciation</b> Students extract meaning from fiction and non-fiction resources in a variety of formats. They demonstrate an enjoyment of reading, including an appreciation of literature and other creative expressions.</p> <p><b>Standard 5: Personal Management</b> Students display evidence of ethical, legal, and social responsibility in regard to information resources and project and self-management.</p>
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<p><b><u>Enduring Understandings</u></b></p> <p>Point estimates of population parameters are usually close but rarely equal to the actual value of the parameter that they are estimating</p> <p>When reading survey results, it is important to always question the sample size, the sampling technique, the questions asked, and the wording used in those questions</p> <p>Meaningful estimates are made by specifying an interval of values on a number line, along with a confidence statement</p> <p>The level of confidence <math>c</math> is the area under the standard normal curve between critical values.</p> <p>A large sample size tends to provide a narrower confidence interval for the same level of confidence.</p> <p>There are two critical values for each level of confidence which can be used to construct confidence intervals for population variance and standard deviation.</p>	<p><b><u>Essential Questions</u></b></p> <ul style="list-style-type: none"> <li>➤ How can you use sample statistics to estimate the value of an unknown population parameter?</li> <li>➤ How do you construct a confidence interval?</li> <li>➤ How do you improve the precision of an estimate without decreasing the level of confidence?</li> <li>➤ How does the t-distribution aid in the construction of confidence intervals?</li> <li>➤ How do you estimate a population proportion using confidence intervals?</li> <li>➤ How do we use the chi-square distribution to construct a confidence interval for the variance and the standard deviation?</li> </ul>
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<b>Key Vocabulary</b>		
<ul style="list-style-type: none"> <li>➤ Point estimate, interval estimate, level of confidence, critical values, sampling error, margin of error, confidence interval, t-distribution, degrees of freedom, population proportion, and chi-square distribution.</li> </ul>		
<b>Learning Objectives / Grade Level Expectations</b>		
<i>Students will:</i>		
<ul style="list-style-type: none"> <li>➤ find a point estimate and a margin of error</li> <li>➤ construct and interpret confidence intervals for the population mean</li> <li>➤ determine the minimum sample size required when estimating <math>\mu</math></li> <li>➤ interpret the <math>t</math>-distribution and use a <math>t</math>-distribution table</li> <li>➤ construct confidence intervals when <math>n &lt; 30</math>, the population is normally distributed, and <math>\sigma</math> is unknown</li> <li>➤ find a point estimate for the population proportion</li> <li>➤ construct a confidence interval for a population proportion</li> <li>➤ determine the minimum sample size required when estimating a population proportion</li> <li>➤ interpret the chi-square distribution and use chi-square distribution table</li> <li>➤ use the chi-square distribution to construct a confidence interval for the variance and the standard deviation</li> </ul>		
<b>ASSESSMENT PLAN</b>		
<b>Summative Assessment(s)/Performance Based Assessments including 21<sup>st</sup> Century Learning</b> <ul style="list-style-type: none"> <li>➤ Quiz After Section 6.2 &amp; 6.4</li> <li>➤ Chapter 6 assessment</li> </ul>	<b>Formative and Diagnostic Assessment(s)</b> <ul style="list-style-type: none"> <li>➤ Common Formative Assessment based on Unit 6, Confidence Intervals</li> <li>➤ Construct confidence intervals and determine margins of error</li> <li>➤ Use <math>t</math>-distributions to construct confidence intervals and construct them for population variance and population standard deviation.</li> <li>➤ Verbal assessments</li> <li>➤ Informal assessments of class work</li> <li>➤ Weekly quiz</li> <li>➤ Homework review</li> </ul>	
<b>LEARNING PLAN COMPONENTS</b>		
<b>Textbook</b>	<b>Supplemental Materials</b>	<b>Suggested Homework and Practice</b>
Larson & Farber Elementary Statistics Fourth Edition	<ul style="list-style-type: none"> <li>➤ Prentice Hall Mathematics – Elementary Statistics Chapter</li> </ul>	Real world problems from the book. .
<b>LEARNING RESOURCES</b>		
<ul style="list-style-type: none"> <li>➤ Prentice Hall Mathematics – Elementary Statistics Chapters</li> <li>➤ Differentiation for Special Needs students</li> <li>➤ <a href="http://ccssmath.org/">http://ccssmath.org/</a></li> <li>➤ <a href="http://illustrativemathematics.org/">http://illustrativemathematics.org/</a></li> <li>➤ <a href="http://commoncoretools.me/2011/01/16/the-illustrative-mathematics-project/">http://commoncoretools.me/2011/01/16/the-illustrative-mathematics-project/</a></li> <li>➤ <a href="https://www.teachingchannel.org/videos/technology-and-math">https://www.teachingchannel.org/videos/technology-and-math</a></li> <li>➤ <a href="http://www.stats.gla.ac.uk/steps/glossary/">http://www.stats.gla.ac.uk/steps/glossary/</a></li> <li>➤ <a href="http://www.apus.edu/media/mathWV/statistics-bk.htm">http://www.apus.edu/media/mathWV/statistics-bk.htm</a></li> <li>➤ <a href="http://www.stattrek.com">http://www.stattrek.com</a></li> <li>➤ <a href="http://www.learner.org/resources/series65.html">http://www.learner.org/resources/series65.html</a></li> <li>➤ <a href="http://davidmlane.com/hyperstat/index.html">http://davidmlane.com/hyperstat/index.html</a></li> </ul>		

**STATISTICS 2 UNIT PLANS # 1 - 4**  
**Stratford Public Schools Statistics**  
**Unit # 2**

**Unit Name:** Chapter 7 *Hypothesis Testing with One Sample*

**Est. # of Weeks:** 4.5 weeks

**In this unit the focus will be on:**

- **Introduction to Hypothesis Testing**
- **Hypothesis Testing for the Mean**
  - Large Samples
  - Small Samples
- **Hypothesis Testing for Proportions**
- **Hypothesis Testing for Variance and Standard Deviation**

**STUDENT LEARNING GOALS**

**CCSS Math Standards**

**CCSS.MATH.CONTENT.HSS.IC.A.1**

Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

**CCSS.MATH.CONTENT.HSS.IC.B.3**

Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

**CCSS.MATH.CONTENT.HSS.IC.B.4**

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

**CCSS.MATH.CONTENT.HSS.IC.B.6**

Evaluate reports based on data.

**CCSS.MATH.CONTENT.HSS.ID.A.2**

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more data sets..

**CCSS.MATH.CONTENT.HSS.ID.A.3**

Interpret differences in shape, center, and spread in the context of the datasets, accounting for possible effects of extreme data points (outliers).

**CCSS.MATH.CONTENT.HSS.ID.A.4**

Use the mean and standard deviation of the data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

**21<sup>st</sup> Century Skills and Expectations**

**Rubric: Critical Skills**

1. Use real-world digital and other research tools to access, evaluate, and effectively apply information appropriate for authentic tasks.
2. Work independently and collaboratively to solve problems and accomplish goals.
3. Communicate information clearly and effectively using a variety of tools/media in varied contexts for a variety of purposes.
4. Demonstrate innovation, flexibility, and adaptability in thinking patterns, work habits, and working/learning conditions.
5. Effectively apply the analysis, synthesis, and evaluative processes that enable productive problem solving.
6. Value and demonstrate personal responsibility, character, cultural understanding and ethical behavior.

<p><u>Math Practices</u></p> <p><a href="#">CCSS.Math.Practice.MP1</a> Make sense of problems and persevere in solving them.</p> <p><a href="#">CCSS.Math.Practice.MP2</a> Reason abstractly and quantitatively.</p> <p><a href="#">CCSS.Math.Practice.MP3</a> Construct viable arguments and critique the reasoning of others.</p> <p><a href="#">CCSS.Math.Practice.MP4</a> Model with mathematics.</p> <p><a href="#">CCSS.Math.Practice.MP5</a> Use appropriate tools strategically.</p> <p><a href="#">CCSS.Math.Practice.MP6</a> Attend to precision.</p> <p><a href="#">CCSS.Math.Practice.MP7</a> Look for and make use of structure.</p> <p><a href="#">CCSS.Math.Practice.MP8</a> Look for and express regularity in repeated reasoning.</p>	<p><u>Interdisciplinary Standards (Technology Integration)</u></p> <p><b>Standard 1: Information Strategies</b> Students determine their need for information and apply strategies to select, locate, and access information resources.</p> <p><b>Standard 2: Information Use</b> Students evaluate, analyze, and synthesize information and data to solve problems, conduct research, and pursue personal interests.</p> <p><b>Standard 3: Information and Technology Application</b> Students use appropriate technologies to create written, visual, oral and multimedia products that communicate ideas and information.</p> <p><b>Standard 4: Literacy and Literary Appreciation</b> Students extract meaning from fiction and non-fiction resources in a variety of formats. They demonstrate an enjoyment of reading, including an appreciation of literature and other creative expressions.</p> <p><b>Standard 5: Personal Management</b> Students display evidence of ethical, legal, and social responsibility in regard to information resources and project and self-management.</p>
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<p><b>Enduring Understandings</b></p> <p>Hypothesis testing is a process that s sample data to test a claim about population parameter</p> <p>Either the null hypothesis or the alternative hypothesis must be true and may represent the original claim.</p> <p>When the standardized test statistic falls in the rejection region, the observed probability (<i>P</i>-value) Of a type I error is less than <math>\alpha</math></p> <p>When the sample size is at least 30, the sampling distribution of the sample means has a normal shape and the mean is the hypothesized mean.</p> <p>If a test statistic falls in a rejection region, it would be considered an unusual event.</p> <p>The <i>t</i>-test for a mean can be used when the population is normal or nearly normal, <math>\sigma</math> is unknown and <math>n &lt; 30</math></p> <p>A hypothesis test for a proportion <i>p</i> can be performed using <i>P</i>-values.</p> <p>If the population is normal, the <math>\chi^2</math>-test can be used to test the variance or the standard deviation.</p>	<p><b>Essential Questions</b></p> <ul style="list-style-type: none"> <li>➤ How can you identify type I and type II errors and interpret the level of significance?</li> <li>➤ How do you know whether to use a one-tailed or a two-tailed statistical test and finding a <i>p</i>-value?</li> <li>➤ How do you determine the <i>null</i> hypothesis and the <i>alternative</i> hypothesis?</li> <li>➤ How do you find critical values for one-tailed and two tailed tests?</li> <li>➤ How do you use the <i>t</i>-test to test a mean <math>\mu</math>?</li> <li>➤ How do you find critical values for a <math>\chi^2</math>-test?</li> <li>➤ How do you use a hypothesis test for standard deviation or population variance?</li> </ul>
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## Key Vocabulary

- Hypothesis test, statistical hypothesis, null hypothesis, alternative hypothesis, type I error, type II error, level of significance, P-value (probability value), left-tailed test, right-tailed test, two-tailed test, z-test for a mean, test statistic, standardized test statistic, rejection region(critical region), t-test for a mean, z-test for a proportion,  $\chi^2$ -test for variance or standard deviation,

## Learning Objectives / Grade Level Expectations

*Students will:*

- state a null hypothesis and an alternative hypothesis
- identify type I and type II errors
- know whether to use a one-tailed or a two-tailed statistical test
- interpret a decision based on the results of a statistical test
- find critical values for a z-test
- use rejection regions for a z-test
- find P-values and use them to test a mean  $\mu$
- find critical values in a t-distribution
- use the t-test to test a mean  $\mu$
- use technology to find P-values and use them with a t-test to test a mean  $\mu$
- use the z-test to test a population proportion  $p$
- find critical values for a  $\chi^2$ -test
- use the  $\chi^2$ -test to test a variance or a standard deviation

## ASSESSMENT PLAN

### Summative Assessment(s)/Performance Based Assessments including 21<sup>st</sup> Century Learning

- Quiz After Section 7.2 & 7.5
- Chapter 7 assessment

### Formative and Diagnostic Assessment(s)

- Common Formative Assessment based on Unit 7, Hypothesis Testing
- Write hypotheses and specify significance
- Find standardized test statistics.
- Verbal assessments
- Informal assessments of class work
- Weekly quiz
- Homework review

## LEARNING PLAN COMPONENTS

Textbook	Supplemental Materials	Suggested Homework and Practice
Larson & Farber Elementary Statistics Fourth Edition	➤ Prentice Hall Mathematics – Elementary Statistics Chapter	Real world problems from the book. .

## LEARNING RESOURCES

- Prentice Hall Mathematics – Elementary Statistics Chapters
- Differentiation for Special Needs students
- <http://ccssmath.org/>
- <http://illustrativemathematics.org/>
- <http://commoncoretools.me/2011/01/16/the-illustrative-mathematics-project/>
- <https://www.teachingchannel.org/videos/technology-and-math>  
<http://www.stats.gla.ac.uk/steps/glossary/>  
<http://www.apus.edu/media/mathWV/statistics-bk.htm>  
<http://www.stattrek.com>  
<http://www.learner.org/resources/series65.html>  
<http://davidmlane.com/hyperstat/index.html>

**STATISTICS 2 UNIT PLANS # 1 - 4**  
**Stratford Public Schools Statistics**  
**Unit # 3**

**Unit Name:** Chapter 8 *Hypothesis Testing with Two Samples*

**Est. # of Weeks:** 5 weeks

**In this unit the focus will be on:**

- Testing the Difference Between Means (Large Independent Samples)
- Testing the Difference Between Means (Small Independent Samples)
- Testing the Difference Between Means (Dependent Samples)
- Testing the Difference Between Proportions

**STUDENT LEARNING GOALS**

**CCSS Math Standards**

**[CCSS.MATH.CONTENT.HSS.ID.B.5](#)**

Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

**[CCSS.MATH.CONTENT.HSS.ID.B.6](#)**

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

**[CCSS.MATH.CONTENT.HSS.IC.A.1](#)**

Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

**[CCSS.MATH.CONTENT.HSS.IC.B.3](#)**

Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

**[CCSS.MATH.CONTENT.HSS.IC.B.4](#)**

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

**[CCSS.MATH.CONTENT.HSS.IC.B.5](#)**

Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

**[CCSS.MATH.CONTENT.HSS.IC.B.6](#)**

Evaluate reports based on data.

**21<sup>st</sup> Century Skills and Expectations**

**Rubric: Critical Skills**

1. Use real-world digital and other research tools to access, evaluate, and effectively apply information appropriate for authentic tasks.
2. Work independently and collaboratively to solve problems and accomplish goals.
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<p><b>Enduring Understandings</b></p> <p>Hypothesis testing uses sample data to decide between two competing claims about a population characteristic.</p> <p>There is a possibility of making a Type I or Type II error when conducting a hypothesis test</p> <p>Tests can be performed using the critical value approach or the p-value approach</p> <p>The level of significance is the total area in the rejection region</p> <p>In a one-tailed hypothesis test, the equivalent confidence level is equal to one minus twice the alpha level.</p> <p>Hypothesis testing for two samples involves the difference between the means or proportions</p> <p>Identifying and labeling each population allows for more accurate and less confusing conclusions</p> <p>Procedures vary for samples that are dependent as opposed to independent</p>	<p><b>Essential Questions</b></p> <ul style="list-style-type: none"> <li>➤ How can you decide whether two samples are independent or dependent?</li> <li>➤ What are the statistical advantages of paired data values? How do we construct statistical tests?</li> <li>➤ How do we compare means from two independent populations when we know <math>\sigma</math> for each population?</li> <li>➤ What if we want to compare means from two independent populations, but we do not know <math>\sigma</math> for each population?</li> <li>➤ How do we use sample data to compare proportions from two independent populations?</li> </ul>
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<p><b>Key Vocabulary</b></p> <ul style="list-style-type: none"> <li>➤ Independent, dependent, paired samples, null hypothesis, alternative hypothesis, two sample z test, test statistic, two sample t</li> </ul>
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test, sampling distribution

### Learning Objectives / Grade Level Expectations

Students will:

- Decide whether two samples are independent or dependent.
- Perform a two-sample z-test for the difference between means.
- Perform a t-test for difference between two population means using small independent samples
- Perform a t-test to test the mean of the differences for a population of paired data
- Perform a z-test for the difference between two population proportions

### ASSESSMENT PLAN

#### Summative Assessment(s)/Performance Based Assessments including 21<sup>st</sup> Century Learning

- Quiz After Section 8.3
- Chapter 8 assessment

#### Formative and Diagnostic Assessment(s)

- Verbal assessments
- Informal assessments of class work
- Quiz after each section
- Homework review
- Chapter Assessment

### LEARNING PLAN COMPONENTS

#### Textbook

Larson & Farber  
Elementary Statistics  
Fourth Edition

#### Supplemental Materials

- Prentice Hall Mathematics –  
Elementary Statistics Chapters

#### Suggested Homework and Practice

Real world problems from the book.  
For example the mean credit card debt  
of males and females in the United  
States.

### LEARNING RESOURCES

- Prentice Hall Mathematics – Elementary Statistics Chapters
- Differentiation for Special Needs students
- <http://ccsmath.org/>
- <http://illustrativemathematics.org/>
- <http://commoncoretools.me/2011/01/16/the-illustrative-mathematics-project/>
- <https://www.teachingchannel.org/videos/technology-and-math>  
<http://www.stats.gla.ac.uk/steps/glossary/>  
<http://www.apus.edu/media/mathWV/statistics-bk.htm>  
<http://www.stattrek.com>  
<http://www.learner.org/resources/series65.html>  
<http://davidmlane.com/hyperstat/index.html>

**STATISTICS 2 UNIT PLANS # 1 - 4**  
**Stratford Public Schools Statistics**  
**Unit # 4**

**Unit Name:** Chapter 9 *Correlation and Regression*

**Est. # of Weeks:** 5 weeks

**In this unit the focus will be on:**

- Correlation
- Linear Regression
- Measures of Regression and Prediction Intervals
- Multiple Regression

**STUDENT LEARNING GOALS**

**CCSS Math Standards**

**CCSS.MATH.CONTENT.HSS.ID.B.5**

Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

**CCSS.MATH.CONTENT.HSS.ID.B.6**

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

**CCSS.MATH.CONTENT.HSS.IC.A.1**

Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

**CCSS.MATH.CONTENT.HSS.IC.B.3**

Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

**CCSS.MATH.CONTENT.HSS.IC.B.4**

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

**CCSS.MATH.CONTENT.HSS.IC.B.5**

Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

**CCSS.MATH.CONTENT.HSS.IC.B.6**

Evaluate reports based on data.

**21<sup>st</sup> Century Skills and Expectations**

**Rubric: Critical Skills**

1. Use real-world digital and other research tools to access, evaluate, and effectively apply information appropriate for authentic tasks.
2. Work independently and collaboratively to solve problems and accomplish goals.
3. Communicate information clearly and effectively using a variety of tools/media in varied contexts for a variety of purposes.
4. Demonstrate innovation, flexibility, and adaptability in thinking patterns, work habits, and working/learning conditions.
5. Effectively apply the analysis, synthesis, and evaluative processes that enable productive problem solving.
6. Value and demonstrate personal responsibility, character, cultural understanding and ethical behavior.

<p><b>Math Practices</b></p> <p><a href="#">CCSS.Math.Practice.MP1</a> Make sense of problems and persevere in solving them.</p> <p><a href="#">CCSS.Math.Practice.MP2</a> Reason abstractly and quantitatively.</p> <p><a href="#">CCSS.Math.Practice.MP3</a> Construct viable arguments and critique the reasoning of others.</p> <p><a href="#">CCSS.Math.Practice.MP4</a> Model with mathematics.</p> <p><a href="#">CCSS.Math.Practice.MP5</a> Use appropriate tools strategically.</p> <p><a href="#">CCSS.Math.Practice.MP6</a> Attend to precision.</p> <p><a href="#">CCSS.Math.Practice.MP7</a> Look for and make use of structure.</p> <p><a href="#">CCSS.Math.Practice.MP8</a> Look for and express regularity in repeated reasoning.</p>	<p><b>Interdisciplinary Standards (Technology Integration)</b></p> <p><b>Standard 1: Information Strategies</b> Students determine their need for information and apply strategies to select, locate, and access information resources.</p> <p><b>Standard 2: Information Use</b> Students evaluate, analyze, and synthesize information and data to solve problems, conduct research, and pursue personal interests.</p> <p><b>Standard 3: Information and Technology Application</b> Students use appropriate technologies to create written, visual, oral and multimedia products that communicate ideas and information.</p> <p><b>Standard 4: Literacy and Literary Appreciation</b> Students extract meaning from fiction and non-fiction resources in a variety of formats. They demonstrate an enjoyment of reading, including an appreciation of literature and other creative expressions.</p> <p><b>Standard 5: Personal Management</b> Students display evidence of ethical, legal, and social responsibility in regard to information resources and project and self-management.</p>
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<p><b>Enduring Understandings</b></p> <p>Bivariate quantitative data can be tested using linear regression hypothesis testing procedures.</p> <p>The strength of a linear relationship can be based on the correlation coefficient.</p>	<p><b>Essential Questions</b></p> <ul style="list-style-type: none"> <li>➤ What are differences between correlation and association when drawing conclusions about data?</li> <li>➤ When is data usable for linear regression hypothesis testing?</li> <li>➤ What are the benefits of performing a linear regression analysis?</li> <li>➤ What are the benefits of performing linear regression analysis?</li> <li>➤ How can the slope of a linear regression equation be interpreted?</li> <li>➤ How can the correlation coefficient of a data set be interpreted?</li> <li>➤ What is the relevance of the correlation coefficient?</li> <li>➤ What is the accuracy of a prediction obtained from a linear regression equation?</li> <li>➤ What is the accuracy of a prediction obtained from a linear regression equation?</li> <li>➤ What is the accuracy of a prediction obtained from a linear regression equation?</li> <li>➤ In what ways can mistakes be made using a linear regression equation to make predictions?</li> </ul>
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<p><b>Key Vocabulary</b></p> <ul style="list-style-type: none"> <li>➤ Correlation, independent variable, dependent variable, scatter plot, correlation coefficient, t-test, test statistic, standardized test</li> </ul>
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statistic, regression line, residuals, total deviation, explained deviation, unexplained deviation, total variance, explained variance, unexplained variance, coefficient of determination, standard error of estimate, bivariate normal distribution, c-prediction interval, multiple regression

**Learning Objectives / Grade Level Expectations**

*Students will:*

- Find a correlation coefficient
- Test a population correlation coefficient  $\rho$  using a table
- Perform a hypothesis test for a population correlation coefficient
- Distinguish between correlation and causation
- Find the equation of a regression line
- Predict y-values using a regression equation
- Find and interpret the coefficient of determination
- Find and interpret the standard error of estimate for a regression line
- Construct and interpret a prediction interval for y
- Use a multiple regression equation to predict y values

**ASSESSMENT PLAN**

**Summative Assessment(s)/Performance Based Assessments including 21<sup>st</sup> Century Learning**

- Quiz After Section 9.2 & 9.4
- Chapter 9 assessment

**Formative and Diagnostic Assessment(s)**

- Verbal assessments
- Informal assessments of class work
- Quiz after each section
- Homework review
- Chapter Assessment

**LEARNING PLAN COMPONENTS**

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- <http://www.stats.gla.ac.uk/steps/glossary/>
- <http://www.apus.edu/media/mathWV/statistics-bk.htm>
- <http://www.stattrek.com>
- <http://www.learner.org/resources/series65.html>
- <http://davidmlane.com/hyperstat/index.html>

# **Appendix: Standards and Rubrics**

## **District of Stratford CCSSM**

### **Overview:**

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+). All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories:

- **Number and Quantity**
- **Algebra**
- **Functions**
- **Modeling**
- **Geometry**
- **Statistics and Probability**

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol

(★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

## Number and Quantity

### **The Real Number System N –RN**

#### **Extend the properties of exponents to rational exponents.**

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.  
*For example, we define  $5^{1/3}$  to be the cube root of 5 because we want  $(5^{1/3})^3 = (5^{1/3})^3$  to hold, so  $(5^{1/3})^3$  must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

#### **Use properties of rational and irrational numbers.**

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

### **Quantities ★ N –Q**

#### **Reason quantitatively and use units to solve problems.**

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

### **The Complex Number System N –CN**

#### **Perform arithmetic operations with complex numbers.**

1. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
2. Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

#### **Represent complex numbers and their operations on the complex plane.**

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,  $(-1 + \sqrt{3}i)^3 = 8$  because  $(-1 + \sqrt{3}i)$  has modulus 2 and argument  $120^\circ$ .*

6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

**Use complex numbers in polynomial identities and equations.**

7. Solve quadratic equations with real coefficients that have complex solutions.
8. (+) Extend polynomial identities to the complex numbers.  
*For example rewrite  $x^2 + 4$  as  $(x + 2i)(x - 2i)$ .*
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

**Vector and Matrix Quantities N –VM**

**Represent and model with vector quantities.**

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g.,  $\mathbf{v}$ ,  $|\mathbf{v}|$ ,  $\|\mathbf{v}\|$ ,  $v$ ).
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

**Perform operations on vectors.**

4. (+) Add and subtract vectors.
- a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- c. Understand vector subtraction  $\mathbf{v} - \mathbf{w}$  as  $\mathbf{v} + (-\mathbf{w})$ , where  $-\mathbf{w}$  is the additive inverse of  $\mathbf{w}$ , with the same magnitude as  $\mathbf{w}$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
5. (+) Multiply a vector by a scalar.
- a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c(v_x, v_y) = (cv_x, cv_y)$ .
- b. Compute the magnitude of a scalar multiple  $c\mathbf{v}$  using  $\|c\mathbf{v}\| = |c|\mathbf{v}$ . Compute the direction of  $c\mathbf{v}$  knowing that when  $|c|\mathbf{v} \neq \mathbf{0}$ , the direction of  $c\mathbf{v}$  is either along  $\mathbf{v}$  (for  $c > 0$ ) or against  $\mathbf{v}$  (for  $c < 0$ ).

**Perform operations on matrices and use matrices in applications.**

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the

matrix has a multiplicative inverse.

11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
12. (+) Work with  $2 \times 2$  matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

# Algebra

## Seeing Structure in Expressions A-SSE

### Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context. ★
  - a. Interpret parts of an expression, such as terms, factors, and coefficients.
  - b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret  $P(1+r)^n$  as the product of  $P$  and a factor not depending on  $P$ .*
2. Use the structure of an expression to identify ways to rewrite it.  
*For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .*

### Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
  - a. Factor a quadratic expression to reveal the zeros of the function it defines.
  - b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
  - c. Use the properties of exponents to transform expressions for exponential functions.  
*For example the expression  $1.15^t$  can be rewritten as  $(1.15^{1/12})^{12t} \approx 1.012^{12t}$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* ★

## Arithmetic with Polynomials and Rational Expressions A -APR

### Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

### Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

### Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships.  
*For example, the polynomial identity  $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$  can be used to generate Pythagorean triples.*
5. (+) Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's

Triangle.(The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)

### **Rewrite rational expressions**

6. Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

### **Creating Equations ★ A -CED**

#### **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .*

### **Reasoning with Equations and Inequalities A -RE I**

#### **Understand solving equations as a process of reasoning and explain the reasoning**

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

#### **Solve equations and inequalities in one variable**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
  - a. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.
  - b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .

### **Solve systems of equations**

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.  
*For example, find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ .*
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater).

### **Represent and solve equations and inequalities graphically**

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

# Functions

## Interpreting Functions F-IF

### Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by  $f(0) = f(1) = 1$ ,  $f(n+1) = f(n) + f(n-1)$  for  $n \geq 1$ .*

### Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.  
*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function. ★*
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

### Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
  - a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
  - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
  - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
  - d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
  - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric

functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  - b. Use the properties of exponents to interpret expressions for exponential functions.  
*For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)^{12t}$ ,  $y = (1.2)^{t/10}$ , and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another say which has the larger maximum.*

## Building Functions F-BF

### Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities. ★
  - a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
  - b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
  - c. (+) Compose functions. *For example, if  $T(y)$  is the temperature in the atmosphere as a function of height, and  $h(t)$  is the height of a weather balloon as a function of time, then  $T(h(t))$  is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★

### Build new functions from existing functions

3. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.  
*Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
  - a. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. *For example,  $f(x) = 2x^3$  or  $f(x) = (x+1)/(x-1)$  for  $x \neq 1$ .*
  - b. (+) Verify by composition that one function is the inverse of another.
  - c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
  - d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

## Linear, Quadratic, and Exponential Models ★ F -LE

### **Construct and compare linear, quadratic, and exponential models and solve problems**

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
  - a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
  - c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to  $ab^{ct} = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.

### **Interpret expressions for functions in terms of the situation they model**

5. Interpret the parameters in a linear or exponential function in terms of a context.

### **Trigonometric Functions F-TF**

#### **Extend the domain of trigonometric functions using the unit circle**

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\pi - x$ ,  $\pi + x$ , and  $2\pi - x$  in terms of their values for  $x$ , where  $x$  is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

#### **Model periodic phenomena with trigonometric functions**

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★

#### **Prove and apply trigonometric identities**

8. Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

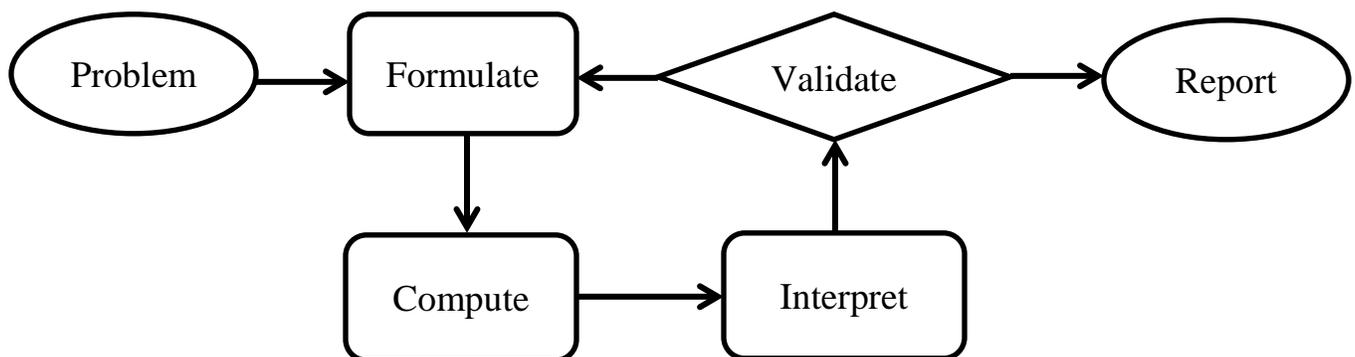
## Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making, expertise, as well as creativity.

- **Estimating** how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- **Planning** a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- **Designing** the layout of the stalls in a school fair so as to raise as much money as possible.
- **Analyzing** stopping distance for a car.
- **Modeling** savings account balance, bacterial colony growth, or investment growth.
- **Engaging** in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- **Analyzing** risk in situations such as extreme sports, pandemics, and terrorism.
- **Relating** population statistics to individual predictions.

1. *identifying variables in the situation and selecting those that represent essential features.*
2. *formulating a model* by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables.
3. *analyzing and performing* operations on these relationships to draw conclusions.
4. *interpreting the results* of the mathematics in terms of the original situation.
5. *validating the conclusions* by comparing them with the situation, and then either improving the model or, if it.
6. *reporting* on the conclusions and the reasoning behind them.

### Modeling Flowchart



# Geometry

## Congruence G-CO

### Experiment with transformations in the plane

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

### Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

### Prove geometric theorem

9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to  $180^\circ$ ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

### **Make geometric constructions**

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

## **Similarity, Right Triangles, and Trigonometry G-SRT**

### **Understand similarity in terms of similarity transformations**

1. Verify experimentally the properties of dilations given by a center and a scale factor:
  - a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
  - b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

### **Prove theorems involving similarity**

4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

### **Define trigonometric ratios and solve problems involving right triangles**

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ★

### **Apply trigonometry to general triangles**

9. (+) Derive the formula  $A = 1/2 ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces)

## Circles G-C

### Understand and apply theorems about circles

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. (+) Construct a tangent line from a point outside a given circle to the circle.

### Find arc lengths and areas of sectors of circles

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

## Expressing Geometric Properties with Equations G-GPE

### Translate between the geometric description and the equation for a conic section

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Derive the equation of a parabola given a focus and directrix.
3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

### Use coordinates to prove simple geometric theorems algebraically

4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point  $(1, \sqrt{3})$  lies on the circle centered at the origin and containing the point  $(0, 2)$*
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★

## Geometric Measurement and Dimension G-GMD

### Explain volume formulas and use them to solve problems

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★

### **Visualize relationships between two-dimensional and three dimensional objects**

4. Identify the shapes of two-dimensional cross-sections of three dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

## **Modeling with Geometry G-MG**

### **Apply geometric concepts in modeling situations**

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

## Statistics and Probability

### Interpreting Categorical and Quantitative Data S-ID

#### Summarize, represent, and interpret data on a single count or measurement variable

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

#### Summarize, represent, and interpret data on two categorical and quantitative variables

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
  - a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose, a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
  - b. Informally assess the fit of a function by plotting and analyzing residuals.
  - c. Fit a linear function for a scatter plot that suggests a linear association.

#### Interpret linear models

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

### Making Inferences and Justifying Conclusions S-IC

#### Understand and evaluate random processes underlying statistical experiments

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

### **Make inferences and justify conclusions from sample surveys, experiments, and observational studies**

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

### **Conditional Probability and the Rules of Probability S-CP**

#### **Understand independence and conditional probability and use them to interpret data**

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
2. Understand that two events  $A$  and  $B$  are independent if the probability of  $A$  and  $B$  occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of  $A$  given  $B$  as  $P(A \text{ and } B)/P(B)$ , and interpret independence of  $A$  and  $B$  as saying that the conditional probability of  $A$  given  $B$  is the same as the probability of  $A$ , the conditional probability of  $B$  given  $A$  is the same as the probability of  $B$ .
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

#### **Use the rules of probability to compute probabilities of compound events in a uniform probability model**

6. Find the conditional probability of  $A$  given  $B$  as the fraction of  $B$ 's outcomes that also belong to  $A$ , and interpret the answer in terms of the model.
7. Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.
8. (+) Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in terms of the model.
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

## Using Probability to Make Decisions S-MD

### Calculate expected values and use them to solve problems

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

### Use probability to evaluate outcomes of decisions

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
  - a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
  - b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

**21<sup>st</sup> Century Assessment Frameworks**  
***Use Real-World Digital and Other Research Tools***

<b>Item</b>	<i>Excellent - 4</i>	<i>Proficient - 3</i>	<i>Sufficient - 2</i>	<i>Insufficient - 1</i>
<b>Use digital technologies as a tool to access, research, organize, and apply information efficiently (time) and effectively (sources).</b>	Uses a wide variety of sources to retrieve information. Information is relevant and detailed.	Uses the minimum number of sources to retrieve information but most of the information is relevant and contains correct essential facts.	Uses the minimum number of sources to retrieve information, however, some of the information is irrelevant and/or contains errors in essential facts.	Cannot find the minimum number of sources required, and information is lacking relevance and essential details.
	Little or no off task behavior - there is a significant return on the time invested.	Quality work is completed efficiently - little wasted time or distraction is evident.	Tasks are completed in a reasonable amount of time - although there is some off task behavior, it is reasonable given the work to be done.	Takes an inordinate amount of time to complete (if at all) basic, routine tasks..
	Identifies a wide variety of types of information needed, locates numerous possible sources and provides a detailed rationale for the selection of these sources.	Identifies a wide variety of types of information needed, locates possible sources and provides a rationale for selection of these sources.	Identifies only minimal information, where to find it and why it is relevant.	Does not identify the information needed, where to find it or why it is relevant.
<b>Manage the flow of information from a wide variety of sources.</b>	Organizes data competently and completely with an effective framework that is well suited to the purpose and the information. Provides a rationale for the framework's use.	Organizes data competently with an effective organizing framework.	Organizes data loosely with some anomalies and evidence of an organizing framework.	Displays little or no evidence of organization.
<b>Evaluate information critically and competently.</b>	Reliably applies, or develops and applies criteria to judge information accuracy or reliability and is able to reflect or improve on the quality of the information.	Reliably applies, or develops and applies, criteria to judge information accuracy or reliability.	Attempts to apply or develop a criteria to judge information accuracy or reliability but application is uneven or unreliable.	Does not apply or develop a criteria to judge information accuracy or reliability.

<b>Item</b>	<i>Excellent - 4</i>	<i>Proficient - 3</i>	<i>Sufficient - 2</i>	<i>Insufficient - 1</i>
<b>Use information accurately and creatively for the issue or problem at hand.</b>	No errors, information is well aligned with purpose and there is strong evidence of original or creative interpretations and applications.	No significant errors, information is well aligned with purpose and there is/are some signs of original or creative interpretations.	While there may be some errors, they do not substantially detract from the purpose – the relationship to the problem is evident.	Information and citations have numerous errors and is at best only tangential to the problem at hand.
<b>Applied solutions/products demonstrate a high degree of alignment between task and solution/product.</b>	Original purpose satisfied completely, goal and result totally aligned.	Original purpose satisfied, goal and result closely aligned.	There is evidence that the original purpose is satisfied, goal and result are at least somewhat aligned.	Original purpose is not satisfied, there is little or no connection between the result and the goal.
<b>Apply a fundamental understanding of the ethical and legal issues surrounding the access and use of information.</b>	Cites all sources using multiple appropriate conventions and formats with no evidence of plagiarism.	Cites all sources using appropriate conventions and formats with no evidence of plagiarism.	Cites all sources appropriately with no evidence of plagiarism.	Rarely or never cites sources or intentionally misrepresents origins of material.

**Passing = 16 with a minimum of 2 in each indicator. 32 is a perfect score.**

## *Work Independently and Collaboratively*

<b>Item</b>	<i>Excellent - 4</i>	<i>Proficient - 3</i>	<i>Sufficient - 2</i>	<i>Insufficient - 1</i>
<b>Works independently when appropriate without being prompted and completes independent work thoroughly.</b>	Meets all deadlines for task completion without adult guidance and there is evidence of early completion and/or time saving strategies.	Meets all deadlines for task completion with minimal adult guidance and with solid work quality.	Even though some adult prompting may be needed, meets most deadlines for task completion with work of acceptable quality.	Cannot complete tasks as designed even with adult intervention and guidance.
<b>Assume shared responsibility for collaborative work, and value the individual contributions made by each team member.</b>	Actively participates in group tasks, is perceived by peers as being a leader and effectively values the contribution of others.	Actively participates in group tasks, is perceived by peers as being helpful and values the contribution of others.	Though prompting may be required, will take part in group tasks with an acceptable level of effort. Does not detract from the group effort.	Does not work toward group goals, may refuse to participate or even detract/impede the group's progress
<b>Listens actively and assists individuals and the group in achieving their goals.</b>	Demonstrates consistent active listening and there is strong evidence of understanding, empathy and response.	Demonstrates some active listening and there is evidence of understanding, empathy and response.	Will passively listen, does not interrupt, and there is some evidence of understanding and response.	Does not or will not listen, may act inappropriately.
	Consistently and effectively helps both individuals and group to achieve their goals. Presence improves the quality of work.	Consistently helps both individuals and group to achieve their goals.	May make some effort to assist others and the group.	Makes no effort to assist others or the group in its work.

<b>Item</b>	<i>Excellent - 4</i>	<i>Proficient - 3</i>	<i>Sufficient - 2</i>	<i>Insufficient - 1</i>
<b>Contributes useful ideas with authentication and encourages others to use their ideas as well.</b>	Contributes useful ideas with authentication and actively encourages others to use their ideas as well.	Contributes useful ideas and can back them up. Occasionally will encourage others to speak up and contribute.	Contributes some useful ideas and will participate.	Does not participate in group interaction even with prompting.
<b>Responds open-mindedly to different ideas and values</b>	There is consistent evidence of open minded approaches – listens and asks respectful questions – seeks to understand new or different ideas. When appropriate will change direction or move to incorporate new learning.	There is evidence of open minded approaches – listens and asks respectful questions – seeks to understand new or different ideas.	While there may not be active support, there are no negative or sarcastic attacks on ideas or values outside of mainstream.	Reacts negatively or sarcastically to ideas or values outside of mainstream.
<b>Applies strategies to improve group efforts and completion of group tasks.</b>	Applies strategies to improve group efforts and completion of the task.	Contributes to improved efforts and completion of tasks	Does his/her share of the group work.	Distracts group members from reaching their goals.
<b>Is willing to undertake a variety of group roles and supports others in their roles as well.</b>	Often assumes a leadership role and supports others in their roles as well. Regardless of role, contribution is positive and constructive.	Assumes various roles including that of leadership when needed.	Assumes various roles as needed or assigned	Does not contribute useful ideas or play any beneficial role;
<b>Understand, negotiate and balance diverse views and beliefs to reach workable solutions, particularly in multi-cultural environments</b>	There is evidence of open minded approaches – listens and asks respectful questions of all parties – seeks to understand how new or different ideas will benefit the group. When appropriate will change direction or move to incorporate new learning.	There is evidence of open minded approaches – listens and asks respectful questions of all parties – seeks to understand how new or different ideas will benefit the group.	While there may not be active support, there are no negative or sarcastic attacks on ideas or values not consistent with personal opinions.	Reacts negatively or sarcastically to ideas not consistent with personal opinions – especially those from other cultures or environments – is a divisive cultural presence.

**Passing = 18 with a minimum of 2 in each indicator. 36 is a perfect score**

## *Communicate Information Clearly and Effectively*

<b>Item</b>	<i>Excellent - 4</i>	<i>Proficient - 3</i>	<i>Sufficient - 2</i>	<i>Insufficient - 1</i>
<b>Listen effectively to decipher meaning, including knowledge, values, attitudes and intentions</b>	Listens with attention and there is strong evidence of understanding of knowledge, meaning, values, attitudes and intentions.	Listens with attention and there is evidence of understanding of knowledge, meaning, and intentions.	Will listen, and while there may be distractions, there is some evidence of understanding of knowledge and meaning.	Does not or will not listen, may act inappropriately.
<b>Articulate thoughts and ideas effectively using oral, written and nonverbal communication skills in a variety of forms and contexts</b>	Presentation clear, logical and organized. Listeners/readers/viewers can follow line of reasoning and the content holds together well.	Presentation is generally clear and well organized. A few minor points may be confusing.	Listeners/readers/viewers can follow presentation with effort. Some arguments are not clear. However, main point is successfully conveyed.	Logic of arguments not clear. Listeners/readers/viewers confused.
	Language/graphical choices are vivid and precise	Language/graphical choices are appropriate; word choices are not particularly vivid or precise	Language/graphical choices are appropriate but may be limited	Language/graphical choices are limited, too complex, or too dull
<b>Utilize multiple technologies, and knows how to judge their effectiveness as well as assess their impact</b>	Uses a variety of techniques and approaches to develop multiple insightful solutions to complex communications arts problems	Uses several techniques or approaches to develop different solutions to a communications problem.	Uses more than one technique or approach to develop different solutions to a communications problem.	Cannot use more than one technique or approach to develop different solutions to a communications problem.

<b>Item</b>	<i>Excellent - 4</i>	<i>Proficient - 3</i>	<i>Sufficient - 2</i>	<i>Insufficient - 1</i>
<p><b>Use a variety of communication tools for a range of purposes (e.g. to inform, instruct, motivate and persuade) and there is evidence of thoughtful selection of strategy to align with message, audience, and purpose.</b></p>	<p>Applies media and techniques that incorporate sophisticated elements and principles of design to effectively or originally communicate personal, cultural, and aesthetic values.</p>	<p>Applies media and techniques that incorporate elements and principles of design to communicate personal, cultural, and aesthetic values.</p>	<p>Applies media and techniques that incorporate the basic elements and principles of design in an attempt to communicate personal, cultural, and aesthetic values.</p>	<p>Is unable to apply media and techniques that incorporate the basic elements and/or principles of design.</p>
	<p>Highly responsive to purpose/ audience comments and needs. It is clear that the audience relates to the communication and the message.</p>	<p>Responsive to purpose/audience comments and needs. Some evidence present that the audience relates to the communication and the message.</p>	<p>Attempts to align and respond to some communication actions and /purpose with audience. There may be some audience response to either the communication or the message.</p>	<p>No connection between communication actions and /purpose, audience comments or needs. Audience may be unresponsive to speaker and message.</p>
<p><b>Communicate effectively in diverse environments (including multi-lingual)</b></p>	<p>Independently pursues several media and integrates experiences in multiple communications pursuits to accomplish even complex multilingual goals.</p>	<p>Consistently contributes through more than one media and tries new avenues.</p>	<p>Occasionally communicates through more than one media; shows some willingness to try new avenues.</p>	<p>Unwilling to communicate through a variety of media.</p>

**Passing = 14 with a minimum of 2 in each indicator. 28 is a perfect score.**

## *Demonstrate Innovation, Flexibility and Adaptability*

<b>Item</b>	<i>Excellent - 4</i>	<i>Proficient - 3</i>	<i>Sufficient - 2</i>	<i>Insufficient - 1</i>
<b>Is aware of their own creative process and the environments in which his/her own thinking is enhanced or inhibited.</b>	Uses their knowledge of their own creative style to broaden their creative range	Clearly demonstrates an awareness of their own creative style and the conditions that contribute to their own creativity.	Articulates an awareness of the creative process in general, and may be able to identify some elements of their own creative style	Does not articulate an awareness of the creative process in general.
<b>Use a wide range of idea creation techniques (such as brainstorming) Create new and worthwhile ideas (both incremental and radical concepts)</b>	Uses a variety of techniques and approaches to develop multiple insightful solutions to a problem or to generate new/creative ideas.	Uses several techniques or approaches to develop different solutions to a problem or to generate new/creative ideas.	Uses more than one technique or approach to develop different solutions to a problem or to generate new/creative ideas.	Cannot use more than one technique or approach to develop different solutions to a problem or to generate new/creative ideas.
<b>Elaborate, refine, analyze and evaluate their own ideas in order to improve and maximize creative efforts</b>	Consistently revisits work and pursues his/her interest in self-evaluation and the process of improving the creativity and originality of one's work	Regularly revisits work. Shows an interest in self-evaluation and the process of improving the creativity and originality of one's work	Sometimes revisits work. Shows an interest in improvement, self-evaluation and the process of producing creative work	Rarely revisits work. Uninterested in improvement, self-evaluation, or the process of producing creative work.

<b>Item</b>	<i>Excellent - 4</i>	<i>Proficient - 3</i>	<i>Sufficient - 2</i>	<i>Insufficient - 1</i>
<b>Be open and responsive to new and diverse perspectives; incorporate group input and feedback into the work</b>	There is consistent evidence of open minded approaches – listens and asks respectful questions – seeks to understand new or different ideas.	There is evidence of open minded approaches – listens and asks respectful questions – seeks to understand new or different ideas.	While there may not be active support, there are no negative or sarcastic attacks on ideas or values outside of mainstream.	Reacts negatively or sarcastically to ideas or values outside of mainstream.
	Actively seeks out feedback and is eager to involve it in the process. When appropriate will change direction or move to incorporate new learning's.	Often will listen and act on feedback from the group.	Occasionally will listen without negativity to outside feedback.	Sees any feedback as negative and defensive reactions dominate the conversation.
<b>Demonstrate originality and inventiveness in work and understand the real world limits to adopting new ideas</b>	Imaginative effort is striking and there is a consistently demonstrated ability to articulate inspiration or genesis of ideas	Imaginative effort is predominant, and there is some ability to articulate inspiration or genesis of ideas.	Derivative elements may be present but there is some evidence of imaginative effort.	Clearly derivative or copied, minimal or no imaginative effort.
	Uses the framework of practical limitations to inform work and challenges conventional thinking, if the problem requires it.	Uses the framework of practical limitations to inform work.	Understands but is not negative regarding limitations parameters.	Uses real world limits as a reason for not trying.
<b>View failure as an opportunity to learn; understand that creativity and innovation is a long-term, cyclical process of small successes and frequent mistakes</b>	Despite setbacks is always positive, is eager to learn, and is able to identify how lessons learned from failure can improve work quality.	Is rarely frustrated, but is otherwise eager to learn, there is evidence that lessons learned from failure have improved work quality.	While may show intermittent frustration, is able to overcome it and remain productive.	Easily frustrated, often negative, and sullen, behavior clearly a barrier to forward progress.

**Passing = 16 with a minimum of 2 in each indicator. 32 is a perfect score.**

## *Analysis, Synthesis, and Evaluative Processes*

<b>Item</b>	<i>Excellent - 4</i>	<i>Proficient - 3</i>	<i>Sufficient - 2</i>	<i>Insufficient - 1</i>
<b>Identify and ask significant questions that clarify various points of view and lead to better solutions</b>	Questions/lines of inquiry are relevant and perceptive and there is evidence of consistent adjustment and response based on context and need.	Most questions/lines of inquiry are relevant and there is evidence of an awareness of the need to adjust questions and approaches based on responses and feedback.	Attempts to generate questions - some of the questions/lines of inquiry are related to the problem at hand.	Either does not formulate questions or the questions generated are irrelevant to the problem at hand.
<b>Analyze how parts of a whole interact with each other to produce overall outcomes in complex systems</b>	Analysis is organized, insightful, relevant and comprehensive. with a complex or elegant framework	Analysis shows an organizational framework/rationale that is consistent and relevant.	Basic analysis is present, data/rationale are loosely connected and there is some attempt to connect parts to whole in a representative framework.	Little or no evidence of analysis and lacks understanding of systems/complexity and the connections of parts to whole.
<b>Effectively analyze and evaluate evidence, arguments, claims and beliefs</b>	Can accurately evaluate explicit information based on criteria that are clear, reasonable, and insightful. Can quickly distinguish between claims, beliefs, and empirical evidence.	Can accurately evaluate explicit information based on criteria that are clear and reasonable. Can distinguish between claims, beliefs, and empirical evidence.	Can evaluate explicit information for relevance or accuracy. While there may be some errors, there is evidence of criteria used in making these judgments.	Is unable to evaluate explicit information for relevance or accuracy.
<b>Synthesize and make connections between information and arguments</b>	Purposefully and thoughtfully incorporates specific information from a wide variety of sources in a synthesis that justifies a point of view.	Thoughtfully incorporates specific information from several sources into a synthesis that justifies a point of view.	Limited sources of information utilized to form stance with evidence of synthesis in support a point of view.	No evidence of synthesis and/or sourcing of information.
	Can identify and support complex and multiple patterns. There is insight and originality in the interpretation.	Can identify multiple patterns - there is evidence of insight and complexity within the connections made.	Can identify superficial or obvious patterns within the information. May attempt more difficult connections with some errors.	No evidence of understanding for connections or patterns within the information.

<b>Item</b>	<i>Excellent - 4</i>	<i>Proficient - 3</i>	<i>Sufficient - 2</i>	<i>Insufficient - 1</i>
<b>Interpret information and draw conclusions based on the best analysis</b>	Articulates a comprehensive/insightful/forceful point of view.	Articulates a thoughtful point of view.	Articulates a rudimentary point of view.	While there may be attempts, a clear point of view is not identifiable.
	Demonstrates the ability to forge an original interpretation based on a variety of sources and materials.	Position has substance and is referenced from multiple sources or points of view.	Takes a position that is one dimensional and is probably derivative of just one or two sources.	Due to the lack of an identifiable position, there is not enough evidence to judge this indicator
<b>Reflect critically on learning experiences and problem solving processes</b>	Throughout the process, constructive and meaningful feedback is consistently sought, considered and discussed.	At key points in the process, feedback is sought, considered and/or discussed	At least once, feedback is sought, considered or discussed.	Feedback is not sought, considered or discussed.
	Consistently revisits work and pursues his/her interest in the process of improving the quality of one's work	Regularly revisits work. Shows an interest in the process of improving the quality of one's work	Sometimes revisits work. Shows an interest in improvement, and the process of producing quality work	Rarely revisits work. Uninterested in improvement, or the process of producing quality work.
	Can speak in depth to personal strengths and weaknesses, regularly applies lessons learned to subsequent experiences. Is able to connect and articulate current and past experiences.	Can speak in depth to personal strengths and weaknesses and regularly applies lessons learned to subsequent experiences. Is able to connect current and past experiences.	Can identify personal strengths and weaknesses, sometimes applies lessons learned to subsequent experiences.	No evidence of improved performance based on prior experience.
<b>Solve different kinds of non-familiar problems in both conventional and innovative ways</b>	Enjoys and seeks out non-conventional problems or issues - is always interested in trying new pathways and strategies.	Without prompting or support will tackle non-conventional problems or issues - is interested and willing to try a new pathway.	With some support can navigate non-conventional problems or issues - will hesitate but can be encouraged to try a new pathway.	Struggles and is easily frustrated by non-conventional problems or issues - goes back to the same process and does not vary approach regardless of circumstances.

**Passing = 22 with a minimum of 2 in each indicator. 44 is a perfect score.**

# STRATFORD DISTRICT HIGH SCHOOLS

## MATHEMATICAL STANDARDS AND RUBRIC

1. Master fundamental mathematical concepts.
2. Communicate mathematical ideas and concepts orally and in writing using a variety of presentation techniques.
3. Apply broad-based curriculum content skills and concepts toward the solution of real world problems.
4. Demonstrate the ability to work effectively both independently and within a group.
5. Apply effective problem-solving strategies and recognize multiple approaches to problem-solving activities.
6. Utilize appropriate technology and technological strategies for research, communication, problem solving and applications.
7. Demonstrate ability to self-assess against established standards.

*This rubric is to be used for all essential questions and understandings in each unit of this course  
(See unit plans for specific questions and understandings).*

SCORE LEVEL	DESCRIPTION
3	The student has demonstrated a <b>full and complete</b> understanding of all concepts and processes essential to this application. The student has addressed the task in a mathematically sound manner. The response contains evidence of the student's competence in problem solving and reasoning, computing and estimating, and communicating to the full extent that these processes apply to the specified task. The response may, however, contain minor arithmetic errors that do not detract from a demonstration of full understanding. Student work is shown or an explanation is included.
2	The student has demonstrated a <b>reasonable</b> understanding of the essential mathematical concepts and processes in this application. The student's response contains most of the attributes of an appropriate response including a mathematically sound approach and evidence of competence with applicable mathematical processes but contains flaws that do not diminish the evidence that the student comprehends the essential mathematical ideas addressed in the task. Such flaws include errors attributed to faulty reading, writing, or drawing skills; errors attributed to insufficient, non-mathematical knowledge; and errors attributed to careless execution of mathematical processes or algorithms.
1	The student has demonstrated a <b>partial</b> understanding of some of the concepts and processes in this application. The student's response contains some of the attributes of an appropriate response but lacks convincing evidence that the student fully comprehends the essential mathematical ideas addressed by this task. Such deficits include evidence of insufficient mathematical knowledge; errors in fundamental mathematical procedures; and other omissions or irregularities that bring into question the extent of the student's ability to solve problems of this general type.
0	The student has demonstrated <b>merely an acquaintance</b> with the topic. The student's response is associated with the task in the item but contains few attributes of an appropriate response. There are significant omissions or irregularities that indicate a lack of comprehension in regard to the mathematical ideas and procedures necessary to adequately address the specified task. No evidence is present to suggest that the student has the ability to solve problems of this general type.