BUNNELL HIGH SCHOOL

AP Calculus BC

Summer Packet

This packet is designed to review Algebra, Geometry, and Pre-Calculus topics which you should be able to handle going into AP Calculus BC.

Topic #1: Solving and Manipulation using Algebra

1.) Simplify: $\dfrac{\frac{3}{x}+\frac{5}{x-4}}{x+3}$

2.) Factor: $(8x^3-27)$

3.) Simplify: $\dfrac{x^{-1}+2}{x^3}$

4.) Factor: $2x^4-6x^3+8x-24$

5.) Solve: $|6+9x|\le 24$

6.) Solve: $2|10b+7|-1>73$

7.) Solve: $|1-4k|<-11$

8.) Solve: $9|r-2|-10<-73$

9.) Simplify: $\dfrac{x^2-16}{9-x}\bullet\dfrac{x^2+x-90}{x^2+14x+40}$

Topic #2: Equations

Part 1: Solve

1.) $-11 + 10(p + 10) = 4 - 5(2p + 11)$

2.) $10(x + 3) - (-9x - 4) = x - 5 + 3$

3.) $-12(x - 12) = -9(1 + 7x)$

4.) $-10n + 3(8 + 8n) = -6(n - 4)$

Solve using a calculator (to three decimal places):

5.) $e^{2x} - 7 = \log_2(x+1)$ (Quadrant 1 solution)

6.) $2x^3 - 3x^2 + 8x + 2 = x^2 - 4$

Write the equation of a line, given the following scenarios:

7.) Passing through (9,1) with a slope of $-\frac{2}{3}$

8.) Passing through (-4,7) and (-3,9)

9.) Passing through (2,-4) and parallel to y=-2x+6

10.) Passing through (-3,5) and perpendicular to 3x+2y=9

11.) Horizontal Line passing through (-8,26)

12.) Passing through (-3,2) and (-3,19)

13.) Passing through the vertex of $y = (x+3)^2 - 4$ and parallel to $y = 3x - 7$

Topic #3: Logarithms

Expand

1) $\log(6 \bullet 11)$

2) $\log\left(6 \bullet 11\right)^5$

3) $\log(3 \cdot 2^3)$

4) $\log\left[(2 \bullet 4)/5\right]$

5) $\log\left(\dfrac{x}{y^6}\right)$

6) $\log(a \bullet b)^2$

Solve:

7.) $\ln(3x+4)=2$

8.) $e^{2y}=4.3$

9.) $\ln(x)=-2.3$

10.) $-10+\log_3(n+4)=-10$

11.) $-2\log_5 7x=2$

12.) $\log_8 2+\log_8 4x^2=1$

13.) $\ln 2-\ln(3x+2)=1$

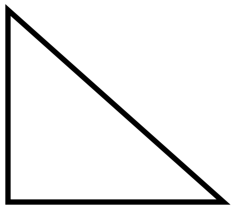
14.) $\ln(x-3)-\ln(x-5)=\ln 5$

Topic #4: Trigonometry

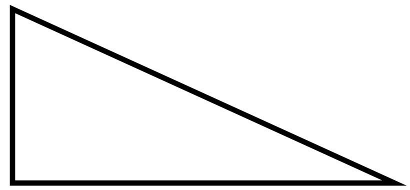
Hint: Make use of the reference sheet at the end of this packet.

Find the missing side lengths in the Special Right Triangles if the hypotenuse is equal to 1. Keep in radical form.

1.) 45-45-90



30-60-90



2.) Use your triangles from above and the ratios below to answer the following in radical form.

$\sin\theta = \dfrac{opposite\ leg}{hypotenuse}$ $\cos\theta = \dfrac{adjacent\ leg}{hypotenuse}$ $\tan\theta = \dfrac{opposite\ leg}{adjacent\ leg}$

a) $\sin(30°)$ = b.) $\cos(30°) =$ c.) $\tan(30°) =$

d.) $\sin(45°) =$ e.) $\cos(45°) =$ f.) $\tan(45°) =$

g.) $\sin(60°) =$ h.) $\cos(60°) =$ i.) $\tan(60°) =$

Use your reference sheet to simplify as much as possible (more than one acceptable answer)

3.) $\cos(7x)\cos(3x) + \sin(7x)\sin(3x)$

4.) $\cos^2(10x) + \sin^2(10x)$

5.) $\sec^2(4x) - \tan^2(4x)$

6.) $\cos^2(4x) - \sin^2(4x)$

7.) $1 - \sin^2\left(\dfrac{x}{2}\right)$

8.) $\cos(5x)\tan(5x)$

9.) $\sin(3x)\cos(3x)$

10.) $\cos^2\theta$ (use power reducing formula)

Topic #5: Limits

1.) $\lim_{x\to -5}\frac{x^2+3x-10}{x+5}$

2.) $\lim_{x\to 1}\frac{x^2-1}{x-1}$

3.) $\lim_{x\to 0}\frac{\frac{1}{-4+x}+\frac{1}{4}}{x}$

4.) $\lim_{x\to 3}\frac{\sqrt{x+6}-3}{x-3}$

5.) $\lim_{x\to a}\frac{x^2-1}{ax-1}$

6.) $\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}$

7.) $\lim_{x\to 2}\frac{e^x}{x-2}$

8.) $\lim_{x\to 0}\frac{xy+7x-12y}{4y}$

9.) $g(x)=\begin{cases} 4, x<2 \\ 2x+7, x\geq 2 \end{cases}$

a.) $\lim_{x\to 2^-} g(x)$

b.) $\lim_{x\to 2^+} g(x)$

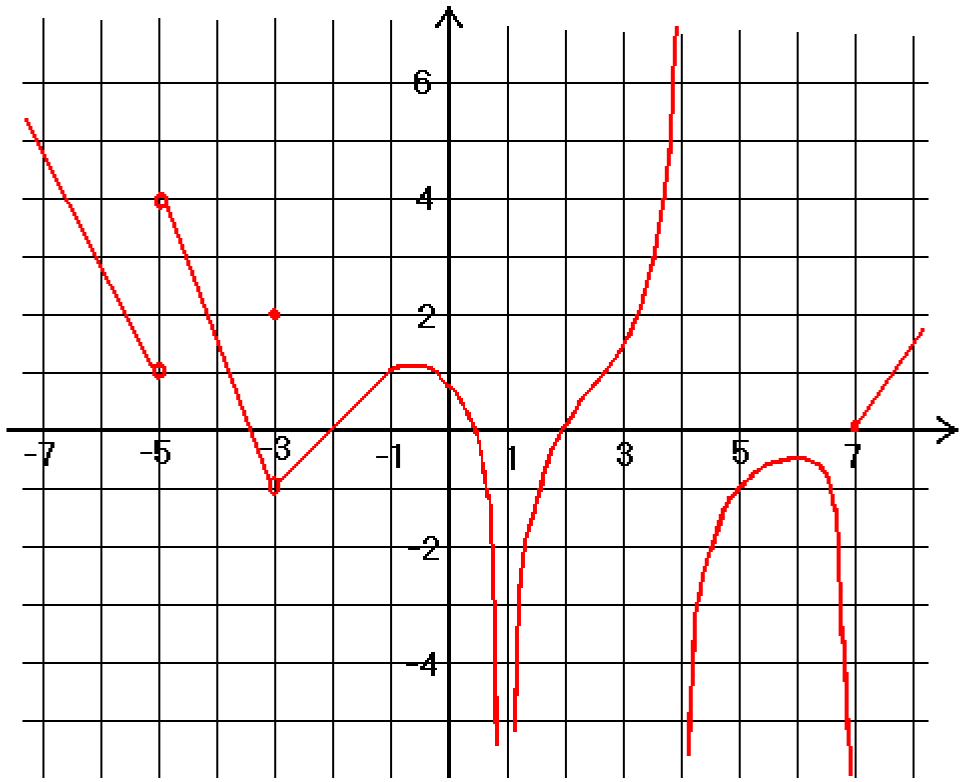
c.) $\lim_{x\to 2} g(x)$

10.) $h(x)=\begin{cases} x^2-2x+1, x<1 \\ x-1, \quad x\geq 1 \end{cases}$

a.) $\lim_{x\to 1^-} h(x)$

b.) $\lim_{x\to 1^+} h(x)$

c.) $\lim_{x\to 1} h(x)$



The function above is f(x). For the function above determine the value of the following limits (if they exist).

11.) $\lim_{x \to -5^-} f(x)$

12.) $\lim_{x \to -5^+} f(x)$

13.) $\lim_{x \to -5} f(x)$

14.) $\lim_{x \to -3^-} f(x)$

15.) $\lim_{x \to -3^+} f(x)$

16.) $\lim_{x \to -3} f(x)$

17.) $\lim_{x \to -1^-} f(x)$

18.) $\lim_{x \to -1^+} f(x)$

19.) $\lim_{x \to -1} f(x)$

20.) $\lim_{x \to 1^-} f(x)$

21.) $\lim_{x \to 1^+} f(x)$

22.) $\lim_{x \to 1} f(x)$

23.) $\lim_{x \to 4^-} f(x)$

24.) $\lim_{x \to 4^+} f(x)$

25.) $\lim_{x \to 4} f(x)$

Topic #6: Continuity

A function is said to be continuous at a point, x = a, if

$$\lim_{x \to a} f(x) = f(a)$$

For the limit to exist, both left and right-hand limits must agree

$$\text{If } \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) \text{ , then } \lim_{x \to a} f(x) \text{ exists}$$

We normally find the value of a constant which makes a function continuous at a point. This most commonly occurs with a piecewise function which is comprised of continuous polynomial functions. The x-value which is the "break" is substituted into both equations and we solve for the variable.

Practical Definition: If you can trace a function without picking up the pencil, then it is continuous.

29. Which of the following functions are continuous for all real numbers x ?

I. $y = x^{\frac{2}{3}}$

II. $y = e^x$

III. $y = \tan x$

(A) None (B) I only (C) II only (D) I and II (E) I and III

42. The graph of which of the following equations has $y = 1$ as an asymptote?

(A) $y = \ln x$ (B) $y = \sin x$ (C) $y = \frac{x}{x+1}$ (D) $y = \frac{x^2}{x-1}$ (E) $y = e^{-x}$

5. Let f be the function defined by the following.

$$f(x) = \begin{cases} \sin x, & x < 0 \\ x^2, & 0 \le x < 1 \\ 2 - x, & 1 \le x < 2 \\ x - 3, & x \ge 2 \end{cases}$$

For what values of x is f NOT continuous?

(A) 0 only (B) 1 only (C) 2 only (D) 0 and 2 only (E) 0, 1, and 2

Topic #7: Sequences and Series

Complete all evens and 29-41 odd. See attached hint sheet in your course email.

EXERCISE SET 10.1

Practice Exercises

In Exercises 1–12, write the first four terms of each sequence whose general term is given.

1. $a_n = 3n + 2$
2. $a_n = 4n - 1$
3. $a_n = 3^n$
4. $a_n = \left(\frac{1}{3}\right)^n$
5. $a_n = (-3)^n$
6. $a_n = \left(-\frac{1}{3}\right)^n$
7. $a_n = (-1)^n(n + 3)$
8. $a_n = (-1)^{n+1}(n + 4)$
9. $a_n = \dfrac{2n}{n + 4}$
10. $a_n = \dfrac{3n}{n + 5}$
11. $a_n = \dfrac{(-1)^{n+1}}{2^n - 1}$
12. $a_n = \dfrac{(-1)^{n+1}}{2^n + 1}$

The sequences in Exercises 13–18 are defined using recursion formulas. Write the first four terms of each sequence.

13. $a_1 = 7$ and $a_n = a_{n-1} + 5$ for $n \geq 2$
14. $a_1 = 12$ and $a_n = a_{n-1} + 4$ for $n \geq 2$
15. $a_1 = 3$ and $a_n = 4a_{n-1}$ for $n \geq 2$
16. $a_1 = 2$ and $a_n = 5a_{n-1}$ for $n \geq 2$
17. $a_1 = 4$ and $a_n = 2a_{n-1} + 3$ for $n \geq 2$
18. $a_1 = 5$ and $a_n = 3a_{n-1} - 1$ for $n \geq 2$

In Exercises 19–22, the general term of a sequence is given and involves a factorial. Write the first four terms of each sequence.

19. $a_n = \dfrac{n^2}{n!}$
20. $a_n = \dfrac{(n + 1)!}{n^2}$
21. $a_n = 2(n + 1)!$
22. $a_n = -2(n - 1)!$

In Exercises 23–28, evaluate each factorial expression.

23. $\dfrac{17!}{15!}$
24. $\dfrac{18!}{16!}$
25. $\dfrac{16!}{2!14!}$
26. $\dfrac{20!}{2!18!}$
27. $\dfrac{(n + 2)!}{n!}$
28. $\dfrac{(2n + 1)!}{(2n)!}$

In Exercises 29–42, find each indicated sum.

29. $\sum_{i=1}^{6} 5i$
30. $\sum_{i=1}^{6} 7i$
31. $\sum_{i=1}^{4} 2i^2$
32. $\sum_{i=1}^{5} i^3$
33. $\sum_{k=1}^{5} k(k + 4)$
34. $\sum_{k=1}^{4} (k - 3)(k + 2)$
35. $\sum_{i=1}^{4} \left(-\frac{1}{2}\right)^i$
36. $\sum_{i=2}^{4} \left(-\frac{1}{3}\right)^i$
37. $\sum_{i=5}^{9} 11$
38. $\sum_{i=3}^{7} 12$
39. $\sum_{i=0}^{4} \dfrac{(-1)^i}{i!}$
40. $\sum_{i=0}^{4} \dfrac{(-1)^{i+1}}{(i + 1)!}$
41. $\sum_{i=1}^{5} \dfrac{i!}{(i - 1)!}$
42. $\sum_{i=1}^{5} \dfrac{(i + 2)!}{i!}$

In Exercises 43–54, express each sum using summation notation. Use 1 as the lower limit of summation and i for the index of summation.

43. $1^2 + 2^2 + 3^2 + \cdots + 15^2$
44. $1^4 + 2^4 + 3^4 + \cdots + 12^4$
45. $2 + 2^2 + 2^3 + \cdots + 2^{11}$
46. $5 + 5^2 + 5^3 + \cdots + 5^{12}$
47. $1 + 2 + 3 + \cdots + 30$
48. $1 + 2 + 3 + \cdots + 40$
49. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{14}{14 + 1}$
50. $\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \cdots + \frac{16}{16 + 2}$
51. $4 + \frac{4^2}{2} + \frac{4^3}{3} + \cdots + \frac{4^n}{n}$
52. $\frac{1}{9} + \frac{2}{9^2} + \frac{3}{9^3} + \cdots + \frac{n}{9^n}$

Topic #8: Miscellaneous

Apply the difference quotient $\frac{f(x+h)-f(x)}{h}$ to the following functions:

1.) $f(x)=4$

2.) $f(x)=x+3$

3.) $f(x)=2x+3$

4.) $f(x)=x^2-x$

Find the vertical and horizontal asymptotes for each function (if they exist)

Hint: Simplify First!

5.) $y=\frac{x^2-2x+1}{x^2-3x-4}$

6.) $y=\frac{x^2-9}{x^3+3x^2-18x}$

7.) $y=\frac{2x^3}{x^3-1}$

8.) $y=\frac{x^2-x-6}{x^3-x^2+x-6}$

9-12: Solve the Trigonometric Equations

9.) $4\sin^2 x = 1$

10.) $\cos^2 x + 2\cos x = 3$

11.) $2\sin x \cos x + \sin x = 0$

12.) $\sin^2 x - \cos^2 x = 1$

13-16: Solve each equation:

13.) $\frac{x+1}{3} - \frac{x-1}{2} = 1$

14.) $\frac{60}{x} - \frac{60}{x-5} = \frac{2}{x}$

15.) $\frac{x}{x-2} + \frac{2x}{4-x^2} = \frac{5}{x+2}$

16.) $\frac{2}{x+5} + \frac{1}{x-5} = \frac{16}{x^2-25}$

17-20: Perform the composition of functions, given $f(x) = x^2$, $g(x) = 2x - 1$ and $h(x) = 2^x$

17.) $f(g(2))$

18.) $f(g(x))$

19.) $f(h(x))$

20.) $g(g(x))$

Topic #9 & #10: Parametric and Polar Equations

Be comfortable with the basic concepts you studied in Pre-Calc, that will be sufficient.