AP Calculus AB summer assignment

DUE DATE: FIRST DAY OF SCHOOL. Name:____________________

The purpose of this assignment is to have you practice the mathematical skills necessary to be successful in Calculus AB. All of the skills covered in this packet are skills from Algebra 2 and Pre-Calculus. If you need to, you may use reference materials to assist you and refresh your memory (old notes, textbooks, online resources, etc.). While the graphing calculators will be used in class, there are NO CALCULATORS ALLOWED on this packet. You should be able to do everything without a calculator.

AP Calculus AB is a fast-paced course that is taught at the college level. There is a lot of material in the curriculum that must be covered before the AP exam in May. Therefore, we do not spend a lot of time re-teaching prerequisite skills. This is why you have this packet. Spend some time with it and make sure you are clear on everything covered in the packet so that you can be successful in Calculus.

This assignment will be collected and graded. Be sure to show all appropriate work to support your answers. You may use loose paper to show any required work. In addition, there will be a quiz on this material during the first week of school.

Good Luck!

Formula Sheet

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$ $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

Logarithms:
$y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \frac{\log_b n}{\log_b a}$

Fractional exponent: $\sqrt[b]{x^c} = x^{\frac{c}{b}}$

Negative Exponents: $x^{-n} = 1/x^n$

The Zero Exponent: $x^0 = 1$, for x not equal to 0.

Multiplying Powers

Multiplying Two Powers of the Same Base:
$(x^a)(x^b) = x^{(a+b)}$

Multiplying Powers of Different Bases:
$(xy)^a = (x^a)(y^a)$

Dividing Powers

Dividing Two Powers of the Same Base:
$(x^a)/(x^b) = x^{(a-b)}$

Dividing Powers of Different Bases:
$(x/y)^a = (x^a)/(y^a)$

Slope-intercept form: $y = mx + b$

Point-slope form: $y = m(x - x_1) + y_1$

Standard form: $Ax + By + C = 0$

Name: ________________________________

Show all work-no credit will be awarded for answers missing appropriate work.
**No Calculators!*

SECTION 1: ALGEBRA REVIEW

For questions 1 and 2, solve for y.

1. $\ln y = kt$
2. $\ln(y-1) - 2 = x + \text{ In } x$

Simplify each expression.

3. $\dfrac{(x^2)^3 x}{x^7}$
4. $\sqrt{x} \bullet \sqrt[3]{x} \bullet x^{\frac{1}{6}}$
5. $\dfrac{5(x+h)^2 - 5x^2}{h}$

Simplify by rationalizing the numerator.

Example:

$$\frac{\sqrt{x+4}-2}{x} = \frac{\sqrt{x+4}-2}{x} \bullet \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \frac{x+4-4}{x\left(\sqrt{x+4}+2\right)} = \frac{x}{x\left(\sqrt{x+4}+2\right)} = \frac{1}{\sqrt{x+4}+2}$$

6. $\dfrac{\sqrt{x+9}-3}{x}$

7. $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

Solve each equation for x over the set of real numbers.

8. $2x^4+3x^3-2x^2=0$

10. $|2x-3|=14$

9. $\dfrac{2x-7}{x+1}=\dfrac{2x}{x+4}$

Find the x and the y intercepts

11. $y=\sqrt{16-x^2}$

12. $y^2=x^2-8x+16$

FUNCTIONS

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "*f* of *g* of *x*" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. **Find each.**

13. $f(2) =$

15. $f(t \div 1) =$

17. $g[f(m+2)] =$

14. $g(-3) =$

16. $f[g(-2)] =$

Let $f(x) = \sin(2x)$. **Find each exactly.**

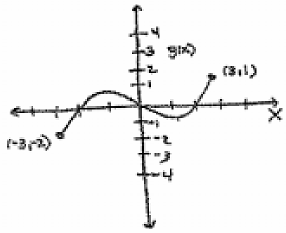
18. $f\left(\frac{\pi}{4}\right) =$ __________

19. $f\left(\frac{2\pi}{3}\right) =$ __________

DOMAIN AND RANGE

Domain – All x values for which a function is defined (input values)
Range – Possible y or Output values

EXAMPLE 1



a) Find Domain & Range of $g(x)$.
The domain is the set of inputs ~~set~~ of the function. Input values run along the horizontal axis. The furthest left input value associated with a pt. on the graph is -3. The furthest right input values associated with a pt. on the graph is 3.
So Domain is $[-3,3]$, that is all reals from -3 to 3.

The range represents the set of output values for the function. Output values run along the vertical axis. The lowest output value of the function is -2. The highest is 1. So the range is $[-2,1]$, all reals from -2 to 1.

EXAMPLE 2

Find the domain and range of $f(x)=\sqrt{4-x^2}$
Write answers in interval notation.

DOMAIN
For $f(x)$ to be defined $4-x^2 \geq 0$.
This is true when $-2 \leq x \leq 2$
Domain: $[-2,2]$

RANGE
The solution to a square root must always be positive thus $f(x)$ must be greater than or equal to 0.
Range: $[0,\infty)$

Find the domain and range of each function.

20. $f(x)=x^2-5$

21. $f(x)=\dfrac{-1}{\sqrt{x+3}}$

22. $f(x)=3\sin x$

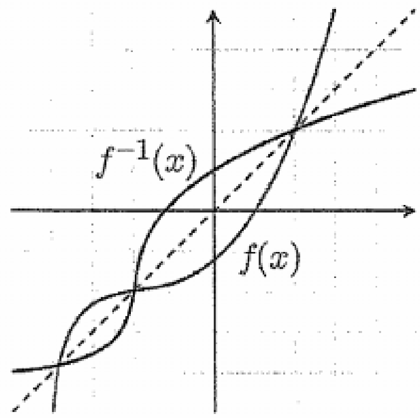
23. $f(x)=\dfrac{2}{x-1}$

INVERSES

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value. Recall $f^{-1}(x)$ is defined as the inverse of $f(x)$

Example 1:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation



Find the inverse for each function.

24. $f(x)=\dfrac{x^2}{3}$

25. $g(x)=\dfrac{x-5}{x-2}$

26. $y=\sqrt{4-x}+1$

27. If the graph of $f(x)$ has the point (2,7) then what is one point that will be on the graph of $f^{-1}(x)$?

28. Explain how the graphs of $f(x)$ and $f^{-1}(x)$ compare.

EQUATION OF A LINE

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$
* LEARN! We will use this formula frequently!

Horizontal line: y = c (slope is 0)

Example: Write a linear equation that has a slope of ½ and passes through the point (2, -6)

Slope intercept form

$y = \frac{1}{2}x + b$	Plug in ½ for m
$-6 = \frac{1}{2}(2) + b$	Plug in the given ordered
$b = -7$	Solve for b
$y = \frac{1}{2}x - 7$	

Point-slope form

$y + 6 = \frac{1}{2}(x - 2)$	Plug in all variables
$y = \frac{1}{2}x - 7$	Solve for y

29. Write the equation of the line tangent to the circle $x^2 + y^2 = 25$ at the point (3,4). Give your equation in point slope form.

UNIT CIRCLE

***You must have these memorized OR know how to calculate their values without the use of a calculator.**

Evaluate each of the following

30. $\sin \pi$
31. $\cos \frac{3\pi}{2}$
32. $\sin\left(-\frac{\pi}{2}\right)$
33. $\sin\left(\frac{5\pi}{4}\right)$
34. $\cos \frac{\pi}{4}$
35. $\cos(-\pi)$
36. $\cos \frac{\pi}{3}$
37. $\sin \frac{5\pi}{6}$
38. $\cos \frac{2\pi}{3}$
39. $\tan \frac{\pi}{4}$
40. $\tan \pi$
41. $\tan \frac{\pi}{3}$
42. $\cos \frac{4\pi}{3}$
43. $\sin \frac{11\pi}{6}$
44. $\tan \frac{7\pi}{6}$
45. $\sin\left(\frac{-\pi}{6}\right)$

INVERSE TRIG. FUNCTIONS

RECALL:

Function	Domain	Range
$y = \arcsin(x) = \sin^{-1} x$	$[-1,1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \arccos(x) = \cos^{-1} x$	$[-1,1]$	$[0, \pi]$
$y = \arctan(x) = \tan^{-1} x$	$(-\infty, \infty)$	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Evaluate the following without a calculator. If necessary, draw a triangle.

46. $\sin^{-1} \frac{1}{2}$

49. $\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

52. $\csc\left(\sin^{-1} \frac{6}{7}\right)$

47. $\sin(\tan^{-1}(-1))$

50. $\cos\left(\cos^{-1} \frac{4}{3}\right)$

53. $\csc\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

48. $\tan^{-1}(\sqrt{3})$

51. $\sin\left(\tan^{-1} \frac{4}{3}\right)$

54. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

55. $\tan^{-1}\frac{\sqrt{3}}{3}$

57. $\sin\left(\arctan\frac{4}{3}\right)$

59. $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$

56. $\sin\left(\cos^{-1}\frac{3}{8}\right)$

58. $cos\left(\arcsin\left(-\frac{2}{3}\right)\right)$

60. $\sin^{-1}\left(\sqrt{3}\sin\frac{\pi}{6}\right)$

61. $\tan\left(\sin^{-1}\frac{1}{2}+2\cos^{-1}\frac{1}{2}\right)$

TRIGONOMETRIC EQUATIONS

Solve each of the equations.

**Recall $\sin^2 x = (\sin x)^2$ AND $x^2 = 25$ then $x = \pm 5$ **

62. $\sin x = -\frac{1}{2}$

63. $2\cos x = \sqrt{3}$

64. $4\sin^2 x = 3$

65. $2\cos^2 x - 1 - \cos x = 0$ *Factor

TRANSFORMATION OF FUNCTIONS

$h(x) = f(x) + c$	Vertical shift c units up	$h(x) = f(x - c)$	Horizontal shift c units right
$h(x) = f(x) - c$	Vertical shift c units down	$h(x) = f(x + c)$	Horizontal shift c units left
$h(x) = -f(x)$	Reflection over the x-axis		

66. Given $f(x) = x^2$ and $g(x) = (-4x + 8)^2 + 7$. How does the graph of $g(x)$ differ from $f(x)$?

67. If the ordered pair (2,4) is on the graph of $f(x)$, find one ordered air that will be on the following functions:

a) $f(x) - 3$

b) $f(x - 3)$

c) $2f(x)$

d) $f(x - 2) + 1$

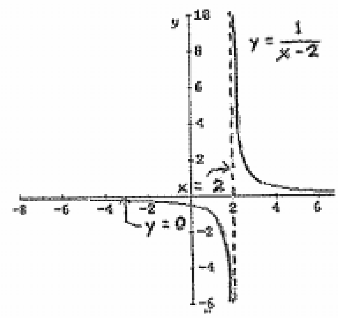
e) $-f(x)$

VERTICAL AND HORIZONTAL ASYMPTOTES

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).
Write a vertical asymptotes as a line in the form x =

Example: Find the vertical asymptote of $y = \frac{1}{x-2}$

Since when $x = 2$ the function is in the form 1/0 then the vertical line $x = 2$ is a vertical asymptote of the function.



Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Example: $y = \frac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Exmaple: $y = \frac{2x^2 + x - 1}{3x^2 + 4}$ (As x becomes very large or very negative the value of this function will approach 2/3). Thus there is a horizontal asymptote at $y = \frac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y = \frac{2x^2 + x - 1}{3x - 3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

Use the functions below to answer each of the following:

a) Find the domain of each.
b) List all of the discontinuities and classify each as a hole or a vertical asymptote.
c) Find the horizontal asymptotes, if any.

68. $f(x) = \dfrac{x-1}{x^2+x-2}$

69. $f(x) = \dfrac{x}{x^2(1-x)}$

70. $f(x) = \dfrac{4-x}{x^2-16}$

71. $f(x) = \dfrac{-3x+1}{\sqrt{x^2+x}}$ *Remember $\sqrt{x^2} = |x|$

EXPONENTIAL FUNCTIONS

Example: Solve for x

$4^{x+1} = \left(\frac{1}{2}\right)^{3x-2}$

$\left(2^2\right)^{x+1} = \left(2^{-1}\right)^{3x-2}$ Get a common base

$2^{2x+2} = 2^{-3x+2}$ Simplify

$2x + 2 = -3x + 2$ Set exponents equal

$x = 0$ Solve for x

Solve for x:

72. $3^{3x+5} = 9^{2x+1}$

73. $(3x+1)^{3/4} = 8$

74. $9x^{2/3} = 4$

75. $\left(\frac{1}{9}\right)^{x} = 27^{2x+4}$

76. $\left(\frac{1}{6}\right)^{x} = 216$

LOGARITHMS

The statement $y = b^x$ can be written as $x = \log_b y$. They mean the same thing.

REMEMBER: A LOGARITHM IS AN EXPONENT

RECALL $\ln x = \log_e x$

Example: Evaluate the following logarithms

$\log_2 8 = ?$

In exponential for this is $2^? = 8$

Therefore $? = 3$

Thus $\log_2 8 = 3$

Evaluate the following logarithms

77. $\log_7 7$

78. $\log_3 27$

79. $\log_2 \frac{1}{32}$

80. $\log_{25} 5$

81. $\log_9 1$

82. $\log_4 8$

83. $\ln \sqrt{e}$

84. $\ln \frac{1}{e}$

PROPERTIES OF LOGARITHMS

$\log_b xy = \log_b x + \log_b y$	$\log_b \frac{x}{y} = \log_b x - \log_b y$	$\log_b x^y = y \log_b x$	$b^{\log_b x} = x$

Examples:

Expand $\log_4 16x$	Condense $\ln y - 2\ln R$	Expand $\log_2 7x^5$
$\log_4 16 + \log_4 x$	$\ln y - \ln R^2$	$\log_2 7 + \log_2 x^5$
$2 + \log_4 x$	$\ln \frac{y}{R^2}$	$\log_2 7 + 5\log_2 x$

Use the properties of logarithms to evaluate the following.

85. $\log_2 2^5$

86. $\ln e^3$

87. $\log_2 8^3$

88. $\log_3 \sqrt[5]{9}$

89. $2^{\log_2 10}$

90. $e^{\ln 8}$

91. $9\ln e^2$

92. $\log_9 9^3$

93. $\log_{10} 25 + \log_{10} 4$

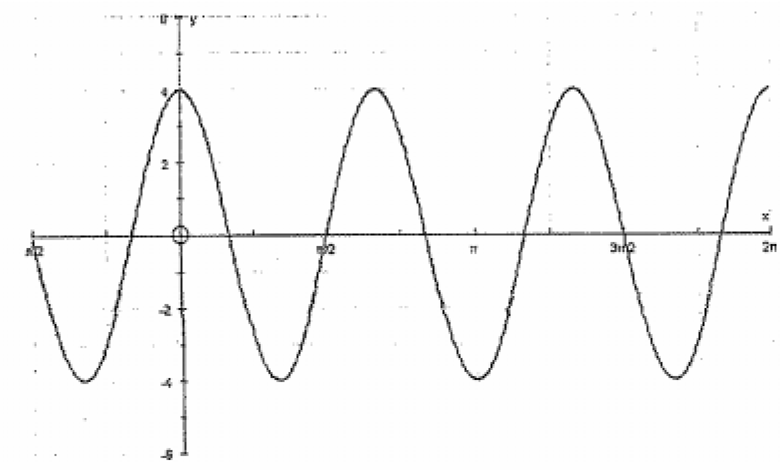
94. $\log_2 40 - \log_2 5$

95. $\log_2 \left(\sqrt{2}\right)^5$

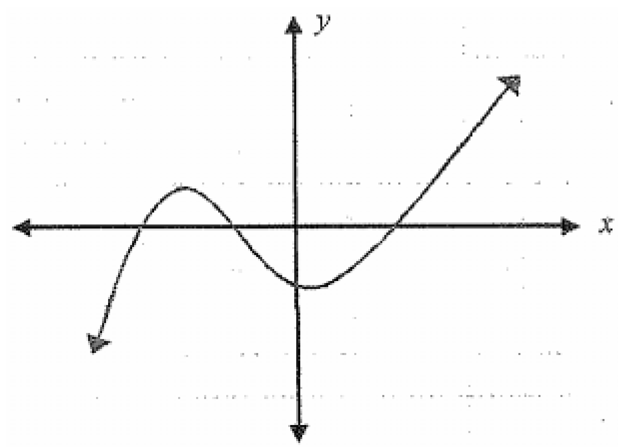
EVEN AND ODD FUNCTIONS

State whether the following functions are even, odd or neither, show ALL work.

96. ______________________



97. ______________________



98. $f(x) = 2x^4 - 5x^2 + x$

99. $g(x) = \dfrac{x^2}{x^4 + 3}$

100. $j\left(x\right) = 2\cos x$

101. $k(x) = \sin x + 4$

EXPONENTIAL MODELS

Exponential Growth Model:
Consider an initial quantity y_0 that changes exponentially with time t. At any time t the amount y units of time may be given by:

$$Y = Y_0 e^{kt}$$

Where k>0 represents the growth rate and k<0 rep0resents the decay rate.

102. Suppose that the cholera bacteria in a colony grow according to the exponential model $P = P_0 e^{kt}$. The colony starts with 1 bacterium and doubles in number every half hour. How many bacteria will the colony contain at the end of 24 hours?

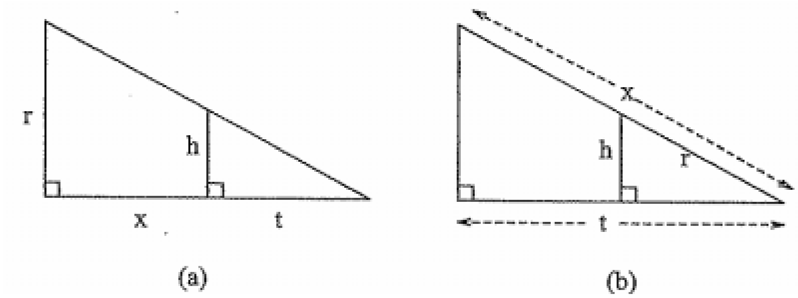
103. A colony of bacteria is grown under the ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 3 hours there are 10,000 bacteria. At the end of 5 hours there are 40,000 bacteria. How many bacteria were present initially?

104. Find the half-life of a radioactive substance with decay equation $Y = Y_0 e^{kt}$ and show that the half-life depends only on k. (set up the equation $\frac{1}{2}Y_0 = Y_0 e^{kt}$ and solve algebraically for t)

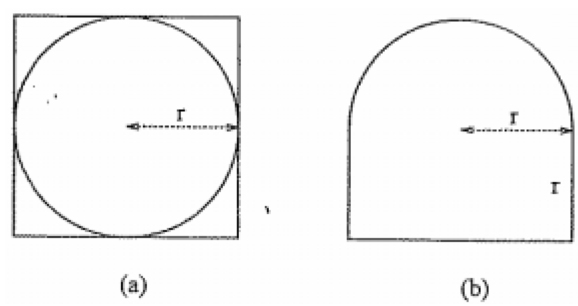
Half-life=____________________

105. Scientists who do carbob-14 dating use 5700 years for its half-life. Find the age of a sample in which 10% of the radioactive nuclei originally present have decayed.

106. Express x in terms of the other variables in the picture.



Use the diagram to answer the next two questions.



107. Find the ratio of the area inside the square but outside the circle to the area of the square in the picture (a) above.

108. Find a formula for the perimeter of a window of the shape in the picture (b) above.

109. A water tank has the shape of a cone (like an ice cream cone without ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle) what is the surface area of the top of the water?

110. Two cars start moving from the same point. One travels south at 100km/hour, the other west at 50km/hour. How far apart are the two hours?

111. A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the horizontal? (Assume that the string is perfectly straight).

FACTORING POLYNOMIALS

What you need to know:

- Perfect Square Trinomials: $a^2 + 2ab + b^2 = (a+b)^2$
  $a^2 - 2ab + b^2 = (a-b)^2$
- Difference of Squares: $a^2 - b^2 = (a+b)(a-b)$
- Sum of Cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- Difference of Cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

PRACTICE PROBLEMS- Factor completely:

112. $3x^2 - x - 10$

113. $4x^2 + 12x - 7$

114. $a^2 - 10a + 25$

115. $4a^2 - 4ab + b^2$

116. $25x^2 - 16y^2$

117. $3x^5 - 48x$

118. $x^6 - y^6$

119. $64 - z^6$

120. $8p^3 + 1$

121. $x(y-2) + 3(2-y)$

122. $x^2 - 6x + 9 - 4y^2$

123. $(x+y)^3 + (x-)^3$

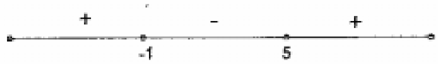
124. $6x^2 - 7xy - 3y^2$

125. $4x^3 + 8x^2y - 5xy^2$

SOLVING POLYNOMIAL INEQUALITTIES

Example 1: $x^2 - 4x - 5 < 0$

Solution: $(x+1)(x-5) < 0$ *factor*

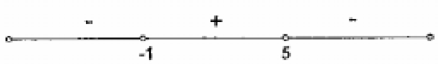


mark the zeros; pick a test point to determine the sign of the polynomial in each interval

The solution set of this **conjunction** is $\{x : -1 < x < 5\}$.

Example 2: $-2x^2 + 8x + 10 < 0$

$-2(x+1)(x-5) < 0$ Factor



Make a sign chart; mark the zeros; pick a test point in each interval and plug it into the factored form to determine the sign of each interval

The solution set of this **disjunction** is: $\{x : x < -1 \text{ or } x > 5\}$

PRACTICE PROBLEMS

State your answer using interval notation.

126. $x^2 - 2x - 8 < 0$

128. $x^3 - 16x > 0$

127. $x^3 + 7x^2 + 10x > 0$

129. $x^2 - x - 12 \le 0$

FUNCTIONS

Find the quotient and the remainder when the first polynomial is divided by the second. No calculators allowed!

130. $x^3 - 2x^2 + 5x + 1,\ x - 1$

131. Determine whether x-1or x+1 are factors of $x^{100} - 4x^{99} + 3$.

Simplify (attention; just perform the indicated operation…)

132. $\dfrac{3}{x^2 - 5x + 6} + \dfrac{2}{x^2 - 4}$

133. $\dfrac{x^2}{x-1} \bullet \dfrac{x+1}{x+2} \div \dfrac{x}{(x-1)(x+2)}$

134. $\dfrac{x^2 - y^2}{\dfrac{x+y}{x^4 - y^4}}$

135. $\dfrac{a^2}{\dfrac{4-5}{4} - \dfrac{a^2}{a-5}}$

136. $\dfrac{4 - x^{-4}}{2 - x^{-2}}$

Solve the fractional equations. If it has no solution, say so…

137. $\frac{3}{x+1}-\frac{1}{x-2}=\frac{1}{x^2-x-2}$

138. $\frac{5}{a^2+a-6}=2-\frac{a-3}{a-2}$

<u>EQUATION CONTAINING RADICALS</u>

Example 1: Solve

$3x-5\sqrt{x}=2$

$3x-2=5\sqrt{x}$ isolate the radical term

$9x^2-12x+4=25x$ square both sides

$9x^2-37x+4=0$ Solve for x

$(x-4)(9x-1)=0$

$x=4$ or $x=\frac{1}{9}$ these are **candidate** solutions!

Now check each candidate above by plugging them into the original equation.

The only possible solution is $x=4$.

Practice Problems: Solve. If an equation has no solution, say so. (Hint: sometimes you may have to square it twice…) CHECK FOR EXTRANEOUS SOLUTIONS.

139. $\sqrt{2x+5}-1=x$

140. $\sqrt{x-1}+\sqrt{x+4}=5$

FUNCTION OPERATIONS AND COMPOSITION: INVERSE FUNCTIONS

What you need to know:

- The **composition** of a function g with a function f is defined as:

$$h(x) = g(f(x))$$

  The domain of h is the set of all x-values such that x is in the domain of f and f(x) is in the domain of g.

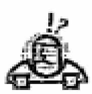
- Functions f and g are inverses of each other provided:

$$f(g(x)) = x \quad and \quad g(f(x)) = x$$

  The function g is denoted as f^{-1}, read as "f inverse".

- When a function f has an inverse, for every point (a,b) on the graph of f, there is a point (b,a) on the graph of f^{-1}. This means that the graph of f^{-1} can be obtained from the graph of f by changing every point (x,y) on f to the point (y,x). This also means that the graphs of f and f^{-1} are reflections about the line y=x. Verify that the functions $f(x) = x^3$ *and* $g(x) = \sqrt[3]{x}$ are inverses!

- Horizontal Line Test: If any horizontal line which is drawn through the graph of a function f intersects the graph no more than once, then f is said to be a **one-to-one** function and has an inverse.



Let's try some problems!

141. Let *f* and *g* be functions whose values are given by the table below. Assume *g* is one-to-one with inverse g^{-1}.

X	f(x)	g(x)
1	6	2
2	9	3
3	10	4
4	-1	6

a) $f(g(3))$

b) $g^{-1}(4)$

c) $f(g^{-1}(6))$

d) $f^{-1}(f(g(2)))$

e) $g(g^{-1}(2))$

Write each absolute value function as a piecewise function.

142. $f(x) = |x+3|$

$f(x) = \begin{cases} \\ \\ \end{cases}$

143. $f(x) = 2|x-1| - 3$

$f(x) = \begin{cases} \\ \\ \end{cases}$

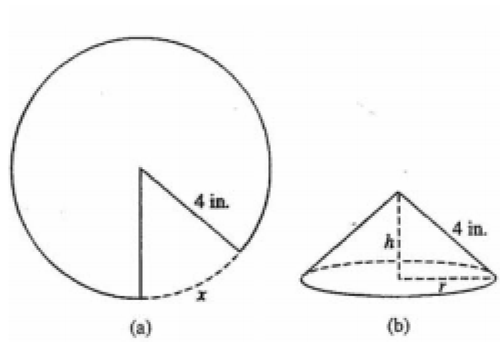
144. $f(x) = |x-4| + 2$

$f(x) = \begin{cases} \\ \\ \end{cases}$

145. $f(x) = \frac{1}{2}|x| + 5$

$f(x) = \begin{cases} \\ \\ \end{cases}$

146. Begin with a circular piece with a 4-in. radius as shown in (a). Cut out a sector with an arc length of x. Join the two edges of the remaining portion to form a cone with radius r and height h, as shown in (b).



a) Explain why the circumference of the base of the cone is $8\pi - x$.

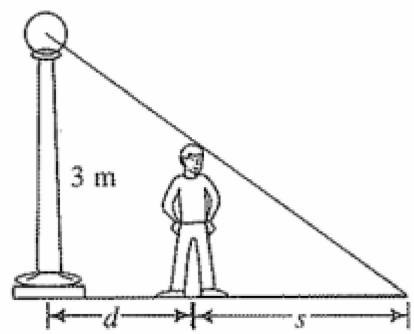
b) Express the radius r as a function of x.

c) Express the height h as a function of x.

d) Express the volume V of the cone as a function of x.

147. The height of a cylinder is twice the diameter. Express the total surface area *A* as a function of the height *h*.

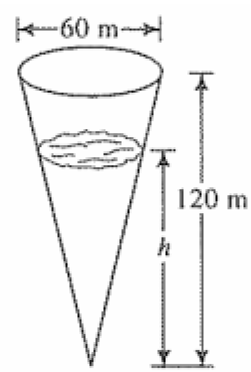
148. A light 3m above the ground causes a boy 1.8m tall to cast a shadow **s** meters long measured along the ground, as shown. Express *s* as a function of *d*, the boy's distance in meters from the light.



Ex. 9

149. Water is flowing at a rate of $5m^3/s$ into the conical tank shown at the right.

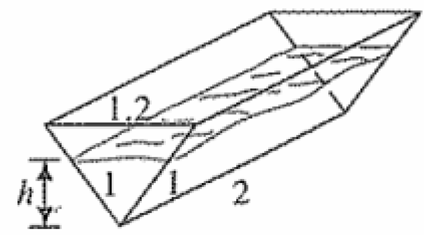
a) Find the volume V of the water as a function of the water level *h*.
b) Find *h* as a function of the time *t* during which water has been flowing into the tank.



Ex. 19

150. A trough is 2m long, and its ends are triangles with sides of length 1m, and 1.2m as shown.

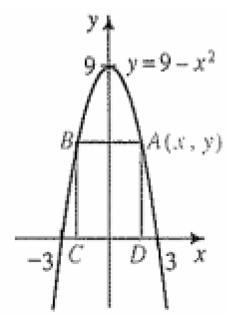
a. Find the volume *V* of the water in the trough as a function of the water level *h*.
b. If water is pumped into the empty trough at the rate of 6 L/min, find the water level h as a function of the time t after the pumping begins. $(1m^3 = 1000L)$



Ex. 20

151. As shown, rectangle ABCD has vertices C and D on the x-axis and vertices A and B on the part of the parabola $y = 9 - x^2$ that is above the *x*-axis.

a) Express the perimeter P of the rectangle as a function of the x-coordinate of A.
b) What is the domain of the perimeter function?
c) For what value of *x* is the perimeter maximum?



Ex. 23